Math 11. Multivariable Calculus.
Written Homework 6.
Due on Wednesday, 10/29/14.

You can turn in this homework by leaving it in the boxes labeled Math 11 in the hallway outside of 008 Kemeny anytime before 3:00 pm on Wednesday.

1. (a) Show that if f is a constant function of two variables (so $f(x, y) = k$ for some constant k) and $R = [a, b] \times [c, d]$, then

$$\iint_R k \, dA = k(b - a)(d - c).$$

   (b) Use part (a) to show that

$$0 \leq \iint_R \sin(\pi x) \cos(\pi y) \, dA \leq \frac{1}{32},$$

   where $R = [0, 1/4] \times [1/4, 1/2]$.

2. Find the average value of $f(x, y) = e^y \sqrt{x + e^y}$ over the rectangle $R = [0, 4] \times [0, 1]$.

3. In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

$$\iint_D f(x, y) \, dA = \int_0^2 \int_0^{\sqrt{y}} f(x, y) \, dx \, dy + \int_2^4 \int_{y-2}^{\sqrt{y}} f(x, y) \, dx \, dy.$$

   Sketch the region D and express the double integral as an iterated integral with reversed order of integration.

4. Sketch the solid whose volume is given by the following iterated integral, and compute the value of that volume:

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx \, dz \, dy.$$