

# Math 11. Multivariable Calculus.

## Written Homework 3.

Due on Wednesday, 10/8/14.

You can turn in this homework by leaving it in the boxes labeled Math 11 in the hallway outside of 008 Kemeny anytime before 3:00 pm on Wednesday.

1. You are told that there is a function  $f = f(x, y)$  whose partial derivatives are  $f_x(x, y) = x + 4y$  and  $f_y(x, y) = 3x - y$ . Should you believe this? Why or why not?
2. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation  $PV = 8.31T$ , where  $P$  is measured in kilopascals,  $V$  in liters, and  $T$  in °K. Use differentials to estimate the approximate change in the pressure if the volume increases from 12 to 12.3 liters and the temperature decreases from 310°K to 305°K.
3. Wheat production in a given year depends on the average temperature  $T$  and the annual rainfall  $R$ . Scientists estimate that the average temperature is rising at a rate of 0.15°C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that at current production levels,  $\frac{\partial W}{\partial T} = -2$  and  $\frac{\partial W}{\partial R} = 8$ .
  - (a) What is the significance of the signs of these partial derivatives?
  - (b) Estimate the current rate of change of wheat production  $dW/dt$ .
4. Suppose that you are hiking a hill whose shape is given by the equation  $z = f(x, y) = 1000 - 0.005x^2 - 0.01y^2$  where  $x, y$  and  $z$  are measured in meters, and you are standing at a point with coordinates  $(60, 40, 966)$ . The positive  $x$  axis points east and the positive  $y$  axis points north.
  - (a) If you walk due south, will you start to ascend or descend? At what rate?
  - (b) If you walk northwest, will you start to ascend or descend? At what rate?
  - (c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?