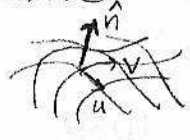
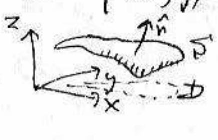
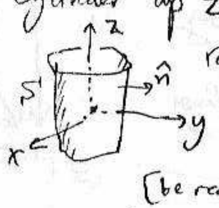
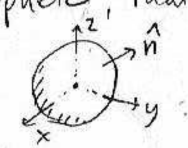
SURFACE INTEGRALS

Calculus III — Alex Barnett

Type of surface S	Parametrize using	oriented area conversion vector from $du\,dv$ to $\vec{dS}$	conversion from $du\,dv$ to dS (scalar)	form of flux $\iint_S \vec{F}\cdot\vec{dS}$, $\vec{F}=(P,Q,R)$
General 	u, v $\begin{cases} x(u,v) \\ y(u,v) \\ z(u,v) \end{cases}$	$\vec{r}_u \times \vec{r}_v$ $\hat{n}$ oriented as shown.	$\lvert \vec{r}_u \times \vec{r}_v \rvert$	$\iint_{\text{domain in } u,v} \vec{F}(\vec{r}(u,v)) \cdot \vec{r}_u \times \vec{r}_v \, du\,dv$ [Note: sometimes $\vec{F}\cdot\hat{n}$ is simple, so can use the surf. int. of this]
'height function' $z = f(x,y)$ 	x, y $\begin{cases} x \\ y \\ z = f(x,y) \end{cases}$	$(-f_x, -f_y, 1)$ $\hat{n}$ upwards	$\sqrt{f_x^2 + f_y^2 + 1}$	useful one: $\iint_D (-Pf_x - Qf_y + R)\,dx\,dy$ D in xy plane; P, Q, R evaluated at $(x, y, f(x,y))$
cylinder up z axis, radius a  [be ready for along y, x axis too]	z, θ $\begin{cases} x = a\cos\theta \\ y = a\sin\theta \\ z \end{cases}$	$(a\cos\theta, a\sin\theta, 0)$ $= a\hat{n}$ $\hat{n}$ outwards.	a	$a \iint_{\text{domain in } z,\theta} \overbrace{(P\cos\theta + Q\sin\theta)}^{\vec{F}\cdot\hat{n}}\, dz\,d\theta$ (don't bother remembering this one!)
sphere, radius a 	ϕ, θ $\begin{cases} x = a\sin\phi\cos\theta \\ y = a\sin\phi\sin\theta \\ z = a\cos\phi \end{cases}$	$a^2 \sin\phi\, \hat{n}$ where $\hat{n} = (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi)$ $\hat{n}$ outwards. Note $\vec{r} = a\hat{n}$.	$a^2 \sin\phi$	$a^2 \iint_{\text{domain in } \phi,\theta} \underbrace{\vec{F}\cdot\hat{n}}_{\text{either evaluate dot product or sometimes it's simple (eg. if } \vec{F} \parallel \hat{n})} \sin\phi \, d\phi\, d\theta$

use for surf integral of scalar