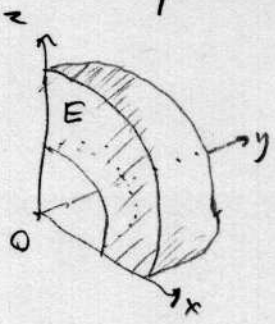
Barnett
11/1/10.

MATH 11 WORKSHEET. Spherical triple integrals.

A) Evaluate $\iiint_E z \, dV$ where E lies between the spheres $x^2+y^2+z^2 = 1$ & $x^2+y^2+z^2 = 4$, in the first octant.



[Hint: what are θ & ϕ ranges?]

B) Find the average distance from a point in a ball of radius a to its center. [Hint: use f = distance func, find $\bar{f}$]

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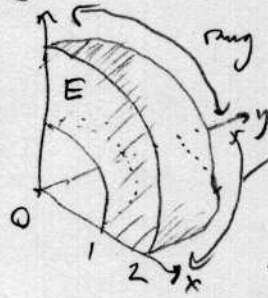
MATH 11 WORKSHEET: Spherical triple integrals.

SOLUTIONS

A) Evaluate $\iiint_E z \, dV$ where E lies between the spheres $x^2+y^2+z^2 = 1$ & $x^2+y^2+z^2 = 4$, in the first octant.

[Hint: what are θ & ϕ ranges?] ← radius 2.



range $0 \le \phi \le \pi/2$.

range $0 \le \theta \le \pi/2$

$f(x,y,z) = z = \rho\cos\phi$

$$I = \int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho\cos\phi \cdot \rho^2\sin\phi \, d\rho\, d\phi\, d\theta$$

↓ can separate

$$= \underbrace{\int_0^{\pi/2} d\theta}_{\pi/2} \cdot \underbrace{\int_0^{\pi/2} \cos\phi\sin\phi\, d\phi}_{\text{either } u=\cos\phi,\ du=-\sin\phi\, d\phi,\ \text{so } -\int_1^0 u\, du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}} \cdot \underbrace{\int_1^2 \rho^3 d\rho}_{\frac{1}{4}\rho^4\Big|_1^2 = \frac{15}{4}}$$

or $\int_0^{\pi/2} \frac{1}{2}\sin 2\phi\, d\phi = -\frac{1}{4}\cos 2\phi\Big|_0^{\pi/2} = \frac{1}{2}$.

$$I = \frac{15\pi}{16}$$

B) Find the average distance from a point in a ball of radius a to its center. [Hint: use f = distance func, find $\bar{f}$]

$f(x,y,z) = \sqrt{x^2+y^2+z^2} = \rho$

Average of func. is $\bar{f} = \dfrac{\iiint_E f\, dV}{\text{Volume of } E}$

E = ball radius a. so Volume $= \frac{4\pi}{3}a^3$.

$$\iiint_E f\, dV = \int_0^{2\pi}\int_0^{\pi}\int_0^a \rho\cdot\rho^2\sin\phi\, d\rho\, d\phi\, d\theta = \underbrace{\int_0^{2\pi} d\theta}_{2\pi} \cdot \underbrace{\int_0^{\pi}\sin\phi\, d\phi}_{2} \cdot \underbrace{\int_0^a \rho^3 d\rho}_{a^4/4}$$

$$= \pi a^4$$

so $\bar{f} = \dfrac{\pi a^4}{(4\pi/3)a^3} = \dfrac{3}{4}a$

On average, points in a ball are distance $3/4$ of the ball's radius from the center.