Barnett
10/20/10

MATH 11 WORKSHEET : Double integrals (rectangle)

A) Let $R = \{(x,y) : 0 \le x \le 2,\ 0 \le y \le 1\}$. Compute $\iint_R x^2 y^3 \, dx\,dy$

Hint: $\int_0^2 x^2 y^3 \, dx \overset{(y \text{ held const})}{=} \quad ?$

so $\int_0^1 \left(\int_0^2 x^2 y^3 \, dx \right) dy = \quad ?$

¿ is there a shortcut here? since y^3 can be brought out...

B) Do the single-variable integral $\int_0^1 \frac{a}{1+ay} \, dy$ for a = some const.
[Hint: $u = 1+ay$]

Use this to find $I = \iint_R \frac{x}{1+xy} \, dx\,dy$
for $R = [0,1] \times [0,1]$

$$= \int_0^1 \underbrace{\left(\int_0^1 \frac{x}{1+xy} \, dy \right)}_{\text{do first to get func of } x} dx$$

What happens if you try doing $\int dx$ first?

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⟶ SOLUTIONS ⟵

A) Let $R = \{(x,y) : 0 \le x \le 2,\ 0 \le y \le 1\}$. Compute $\iint_R x^2 y^3 \, dx\,dy$

Hint: $\displaystyle \int_0^2 x^2 y^3 \, dx \overset{\text{(y held const)}}{=} ?\ y^3 \int_0^2 x^2 \, dx = y^3 \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3} y^3$

so $\displaystyle \int_0^1 \left(\int_0^2 x^2 y^3 \, dx \right) dy = ?\ \int_0^1 \frac{8}{3} y^3 \, dy = \frac{8}{3} \cdot \left[\frac{y^4}{4}\right]_0^1 = \frac{2}{3}.$

Is there a shortcut here? since y^3 can be brought out... simpler is $\int_0^1 y^3 dy \cdot \int_0^2 x^2 dx$.

B) Do the single-variable integral $\displaystyle \int_0^1 \frac{a}{1+ay} \, dy$ for a = some const. [Hint: $u = 1+ay$]

$u = 1 + ay$
$du = a\,dy$. $\displaystyle \frac{1}{a} \int \frac{a}{u} \, du = \int u^{-1} du = \ln|u| \overset{\text{make definite}}{=} \ln|1+ay| \Big|_0^1 = \ln(1+a)$

Use this to find $\displaystyle I = \iint_R \frac{x}{1+xy} \, dx\,dy$
for $R = [0,1] \times [0,1]$

$$= \int_0^1 \underbrace{\left(\int_0^1 \frac{x}{1+xy} \, dy \right)}_{\text{do first to get func of } x.} dx$$

: x plays rôle of a in above, so reuse that.

$= \displaystyle \int_0^1 \ln(1+x) \, dx$ (by parts) $u = \ln(1+x) \to u' = \frac{1}{1+x}$, $v' = 1 \to v = x$

$= \displaystyle x \ln(1+x) \Big|_0^1 - \int_0^1 x \frac{1}{1+x} \, dx$ partial fractions $= \frac{x+1-1}{1+x} = 1 - \frac{1}{1+x}$

$= \ln 2 - \cancel{0 \ln 1} - \left[x - \ln(1+x) \right]_0^1 = 2\ln 2 - 1$

What happens if you try doing $\int dx$ first? It's messier! (but possible) ⇒ choose easiest way always.