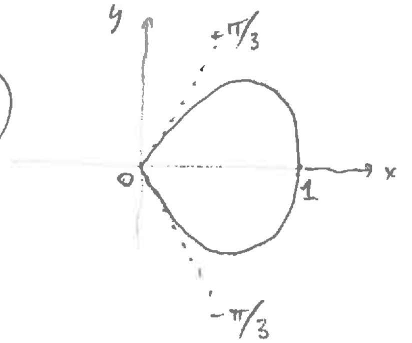
10/25/10
Barnett

MATH 11 WORKSHEET: Polar double integrals.

A)

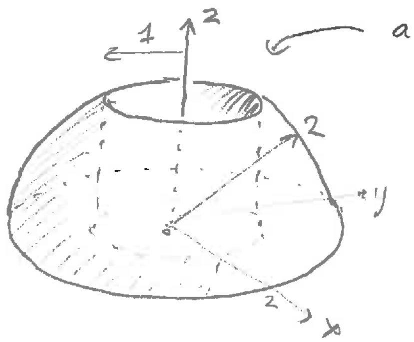


Find the <u>average</u> value of $\sqrt{x^2+y^2}$ (the distance func). in the 'petal' $r = \cos\frac{3\theta}{2}$ shown:

[Hint: $\cos^3 z = \dfrac{\cos 3z + 3\cos z}{4}$]

· Interpret the <u>integral</u> you just did as the volume of a 3d solid.

B) Write a polar integral representing the volume above the plane $z=0$, below the sphere $x^2+y^2+z^2=4$, and outside the cylinder. $x^2+y^2=1$



a hole 'drilled' in a hemisphere.

Compute it! :

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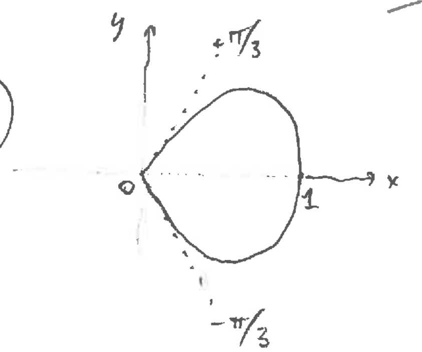
MATH 11 WORKSHEET : Polar double integrals

~~ SOLUTIONS ~~

A)



Find the <u>average</u> value of $\sqrt{x^2+y^2}$ ← $f(x,y)$ (the distance from 0) in the 'petal' $r = \cos\frac{3\theta}{2}$ shown:

$$\text{average value of } f = \frac{\iint_D f(x,y)\,dA}{\text{area of } D}$$

area of D — $\pi/6$ from lecture.

$$I = \iint_D f(x,y)\,dA = \int_{-\pi/3}^{\pi/3} \underbrace{\int_0^{\cos\frac{3\theta}{2}} r \cdot r\,dr}\,d\theta$$

polar factor

$$\frac{r^3}{3}\Big|_0^{\cos\frac{3\theta}{2}} = \frac{1}{3}\cos^3\frac{3\theta}{2}$$

using Hint.

$$= \frac{1}{12}\left(\cos\frac{9\theta}{2} + 3\cos\frac{3\theta}{2}\right)$$

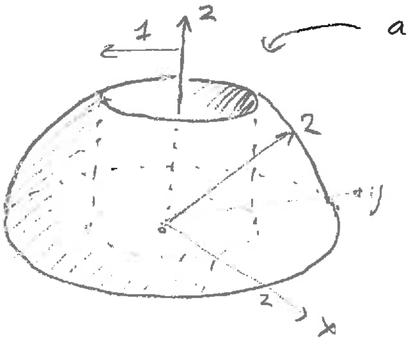
[Hint: $\cos^3 z = \frac{\cos 3z + 3\cos z}{4}$]

$$\text{so } I = \frac{1}{12}\left[\frac{2}{9}\sin\frac{9\theta}{2} + 3\,\frac{2}{3}\sin\frac{3\theta}{2}\right]_{-\pi/3}^{\pi/3} = \frac{1}{6}\left(1-\frac{1}{9}\right)2 = \frac{8}{27}$$

$$\text{avg. } \bar{f} = \frac{I}{\text{area of } D} = \frac{8/27}{\pi/6} = \frac{16}{9\pi} \approx 0.57\ldots \text{ about right}$$

Interpret the <u>integral</u> you just did as the volume of a 3d solid: ~~…~~ I = volume above the petal between plane $z=0$ & cone $z=\sqrt{x^2+y^2}$

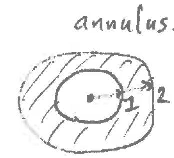
B) Write a polar integral representing the volume above the plane $z=0$, below the sphere $x^2+y^2+z^2=4$, and outside the cylinder $x^2+y^2=1$



← a hole 'drilled' in a hemisphere.

annulus.

$$\text{Vol} = I = \iint_D \sqrt{4-x^2-y^2}\,dA \qquad \text{where } D =$$



$$= \int_0^{2\pi} \underbrace{\int_1^2 \sqrt{4-r^2}\,r\,dr}\,d\theta$$

$\left[\begin{matrix} u = 4-r^2 \\ du = -2r\,dr \end{matrix}\right]$ → $-\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{3}u^{3/2}$

$$= -\frac{1}{3}(4-r^2)^{3/2}\Big|_{r=1}^{r=2} = \frac{3^{3/2}}{3} = \sqrt{3}$$

Compute it! :

$$I = \int_0^{2\pi} \sqrt{3}\,d\theta = 2\pi\sqrt{3}$$

Notice : rotational symmetry so $d\theta$ integral merely gave factor of 2π (easy).