Your name:

Instructor (please circle): Barnett Van Erp

Math 11 Fall 2010: written part of HW9 (due Mon Nov 29)

Please show your work. No credit is given for solutions without justification.

(1) [8 points]

(a) Let $\mathbf{F} = (2xy,\ x^2 + 2yz,\ y^2)$ be a vector field in $\mathbb{R}^3$. Is there a scalar field f such that $\nabla f = \mathbf{F}$? Explain.

(b) Is there a vector field $\mathbf{G}$ such that $\nabla \times \mathbf{G} = \mathbf{F}$? Explain.

(c) Let C be the triangle formed by the boundary of the plane $x + y + z = 1$ restricted to the first octant, traversed in a counter-clockwise sense when viewed in the xy-plane. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$. If you make use of one of your above answers (and we suggest you do), explain how.

(2) [8 points] Let S be the part of the sphere of radius 2 centered at the origin, lying in the region $y \geq 0$, $z \geq 0$. Compute $\iint_S yz \, dS$

(3) [10 points] Let S be the part of the paraboloid $z = x^2 + y^2$ with $z \leq 1$, with surface normal oriented upwards. Let $\mathbf{F}$ be the vector field $(xz,\ yz,\ 1)$

(a) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$

(b) Use this to evaluate $\iint_T \mathbf{F} \cdot d\mathbf{S}$ where T is the *entire* surface of the solid truncated paraboloid bounded by $z = x^2 + y^2$ and $z = 1$, with surface normal everywhere oriented *outwards*: