Your name:

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Math 11 Fall 2010: written part of HW8 (due Wed Nov 17)
Please show your work. No credit is given for solutions without justification.

(1) (a) Consider the vector field

$$\mathbf{F} = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle,$$

and the curve C parametrized as $\mathbf{r}(t) = \langle t, \sin \pi t \rangle$, with $1 \le t \le 2$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. Explain how you got your answer.

(b) The vector field $\mathbf{F}$ is not continuous at $(x, y) = (0, 0)$. Nevertheless, $\mathbf{F}$ is a conservative vector field. Show this directly by finding a potential function $f(x, y)$ for $\mathbf{F}$.

(c) Let C be the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$, with counter clockwise orientation. Observe that the ellipse circles around the singular point $(0,0)$. Find the value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$. Justify your answer.

(2) Let C be the closed curve composed of the segment of the parabola $y = x^2$, for $-2 \leq x \leq 2$, and the straight line segment connecting point $(2,4)$ and $(-2,4)$. Assume that C is oriented counter clockwise. Find the value of the line integral $\oint_C x^2 y\, dx + \ln(y)\, dy$. Explain your solution.

(3) Find the area of the region D that is bounded by the curve $\mathbf{r}(t) = \langle 1 - t^2, t^2 - t^3 \rangle$, with $0 \leq t \leq 1$, and the x-axis. Suggestion: Use Green's Theorem.