Solutions

1	- 8 pts
2	- 10 pts
3	- 10 pts
	- 28 pts.

Your name:

Instructor (please circle): Barnett Van Erp

Math 11 Fall 2010: written part of HW5 (due Wed Oct 27)

Please show your work. No credit is given for answers without justification.

1. [8 points] You are designing a cuboid-shaped aquarium with volume 3 cubic feet. The base is made of slate and the sides are made of glass (the aquarium has no top). If slate costs \$6 per square foot and glass only \$1 per square foot, what is the total material cost of the cheapest aquarium you can make, and what dimensions achieve this?

$V(x,y,z) = \text{Volume} = xyz = 3.$

$\text{Cost} = xy(6) + 2xz(1) + 2yz(1) = F(x,y,z)$

Lagrange Multipliers — gradients must be parallel.

$$\nabla F = \lambda \nabla V$$

$$\langle 6y+2z, 6x+2z, 2x+2y\rangle = \lambda\langle yz, xz, xy\rangle$$

$$\begin{aligned} 6y+2z &= \lambda yz \\ 6x+2z &= \lambda xz \\ 2x+2y &= \lambda xy \end{aligned} \quad\Rightarrow\quad \begin{aligned} 6xy+2xz &= \lambda xyz \quad (1)\\ 6xy+2yz &= \lambda xyz \quad (2)\\ 2xz+2yz &= \lambda xyz \quad (3) \end{aligned}$$

From (1), (2)

$$\begin{aligned} 2xz &= 2yz \\ xz &= yz \quad \text{, since it's true for all } z. \\ x &= y. \end{aligned}$$

~~(1) + (2) - (3)~~ $\cancel{12xy = \lambda xyz}$

$\cancel{12 = \lambda z}$

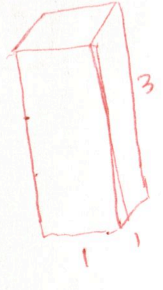
~~sub into (3)~~ $\cancel{4xz = \lambda x^2 z}$

$\cancel{= 3\lambda}$

$$6x^2 = 2xz$$

$$z = 3x$$

$xyz = 3 \Rightarrow 3x^3 = 3$ $\boxed{x=1,\ y=1,\ z=3}$



$F(x,y,z) = 18$

[Unusual design, eh? I wonder why it's not used for real aquaria...]

It would fall over very easily.

Mathematically (economically) efficient but mechanically unstable!

2. [10 points]

7 pts (a) Compute $\iint_R x(1+xy)^4\,dA$ over the rectangle $R = [0,2] \times [0,1]$. [Hint: don't expand out the 4th power!]

$$\int_0^2\int_0^1 x(1+xy)^4\,dy\,dx = \int_0^2\int_{1+(0)x}^{1+(1)x} u^4\,du\,dx$$

$$\boxed{\begin{aligned} u &= 1+xy \\ du &= x\,dy \end{aligned}}$$

$$= \int_0^2 \left[\frac{u^5}{5}\right]_1^{1+x} dx$$

$$= \int_0^2 \left(\frac{(1+x)^5}{5} - \frac{1}{5}\right) dx$$

$$= \left[\frac{(1+x)^6}{30} - \frac{x}{5}\right]_0^2$$

$$= \frac{729-12-1}{30}$$

$$= \frac{358}{15}$$

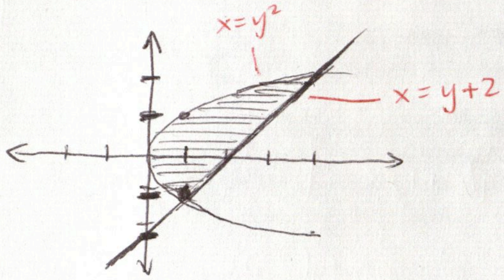
(b) By interpreting the integral as the volume of a solid body, evaluate $\iint_D \sqrt{4-x^2-y^2}\,dA$ where the domain is $D = \{(x,y) : y \geq 0,\ x^2+y^2 \leq 4\}$ [Hint: do not try to evaluate this as an iterated integral. Rather, relate it to a well-known solid body]

3 pts

$\frac{1}{4}$ of a sphere with radius 2.

$$\frac{8\pi}{3}$$

3. [10 points] Compute the integral of the function $f(x, y) = 2x + y$ over the region bounded by the line $y = x - 2$ and curve $x = y^2$



intersect at $x=1$, $x=4$.
$y=-1$, $y=2$

$$\int_{-1}^{2}\int_{y^2}^{y+2} (2x+y)\, dx\, dy$$

$$= \int_{-1}^{2} \left[x^2 + xy\right]_{x=y^2}^{x=y+2} dy$$

$$= \int_{-1}^{2} \left[(y+2)^2 + y(y+2) - y^4 - y^3\right] dy$$

$$= \int_{-1}^{2} \left[(y+2)^2 + 2y + y^2 - y^3 - y^4\right] dy$$

$$= \left[\tfrac{1}{3}(y+2)^3 + y^2 + \tfrac{1}{3}y^3 - \tfrac{1}{4}y^4 - \tfrac{1}{5}y^5\right]_{-1}^{2}$$

$$= \left(\tfrac{64}{3} + \cancel{4} + \tfrac{8}{3} - \cancel{4} - \tfrac{32}{5}\right) - \left(\cancel{\tfrac{1}{3}} + 1 - \cancel{\tfrac{1}{3}} - \tfrac{1}{4} + \tfrac{1}{5}\right)$$

$$= \frac{333}{20}$$