V63.0123-1 : Calculus III. Sample Midterm1

You have 60 minutes. Please find potentially useful equations on back.

1. [10 points]

   In a spaceship an astronaut throws a ball from the location $(1, 1, 1)$ with velocity $(2, -1, -1)$. Because there is no gravity the ball travels in a straight line! A nearby wall of the spaceship is defined by $x - 2y + z - 4 = 0$

   (a) At what location does the ball hit the wall?

   (b) Find the angle between the path of the ball and the surface of the wall (you can leave as a numeric expression to be evaluated).

2. [12 points]

   A three-dimensional object is defined by $x - \sin y \, \cos z = 0$ in the domain $y \in [0, \pi]$ and $z \in [0, \pi/2]$.

   (a) Sketch the object.

   (b) Sketch the curve formed where this object intersects the plane which passes through the origin and is perpendicular to the vector $(0, -1, 1)$.

   (c) Find a parametric equation for this curve.

3. [8 points]

   Find the curvature κ of the curve $y = ax^2$, $z = 0$, at the origin. In which direction is the normal vector **N** at this point?

4. [10 points]

   Given 3 points $A(0, 0), B(0, 10), C(3, 4)$ in $\mathbb{R}^2$, find

   (a) an equation for the line passing A and perpendicular to BC,

   (b) the area of the triangle defined by vertices ABC.

5. [10 points]

   A curve is defined by $x(t) = rt - r \sin t$ and $y(t) = r - r \cos t$, in the domain $t \in [-\pi, \pi]$.

   (a) Find the velocity and speed as a function of t.

   (b) Find the distance along the curve.

   (c) Is the curve smooth? Please explain your answer.

$$
\begin{aligned}
\sin(\theta+\phi) &= \sin\theta\cos\phi+\cos\theta\sin\phi \\
\cos(\theta+\phi) &= \cos\theta\cos\phi-\sin\theta\sin\phi \\
\frac{d}{dx}\sin x &= \cos x \\
\frac{d}{dx}\cos x &= -\sin x \\
\frac{d}{dx}\tan x &= \sec^2 x \\
\frac{d}{dx}\sin^{-1} x &= \frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}\cos^{-1} x &= -\frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}\tan^{-1} x &= \frac{1}{1+x^2} \\
\text{surface area of revolution} &= \int_a^b dt\, 2\pi y(t)\sqrt{x'(t)^2+y'(t)^2} \\
\kappa &= \frac{|\mathbf{T}'|}{|\mathbf{r}'|} = \frac{|\mathbf{r}''\times\mathbf{r}'|}{|\mathbf{r}'|^3} \\
\text{spherical coords:} \quad x &= \rho\sin\phi\cos\theta \\
y &= \rho\sin\phi\sin\theta \\
z &= \rho\cos\phi
\end{aligned}
\tag{1}
$$