Math 11 Fall 2010 Midterm Exam I

Tuesday, October 19, 2010

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PRINT NAME: ______________________________

Instructions: This is a closed book, closed notes exam. You have two hours, do all problems. There are 57 points total, and points for each problem are indicated.

Use of calculators is not permitted.

You must justify all of your answers to receive credit.

The Honor Principle requires that you neither give nor receive any aid on this exam.

GOOD LUCK!

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper.

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1. [10 points] Points $A(0,0,0), B(2,0,0), C(2,2,0), D(0,2,0) E(0,0,2), F(2,0,2), G(2,2,2), H(0,2,2)$ are the eight vertices of a cube.

   (a) Find an equation for the plane through points A, C and H.

   (b) Calculate the surface area of the triangle ΔACH. (Hint: a triangle is one half of a parallellogram.)

(c) Calculate the distance from point D to the plane ACH.

2. [12 points]

(a) Calculate the arc length of the curve

$$\mathbf{r}(t) = \langle 2|t|^{3/2}, \cos t + \sin t, \cos t - \sin t \rangle,$$

for the piece of the curve with $-2/9 \leq t \leq 2/9$.

(b) Find a parametric equation for the tangent line to the curve $\mathbf{r}(t) = \langle e^t, e^{2t}, e^{3t} \rangle$ at the point $(2, 4, 8)$.

(c) Calculate the curvature $\kappa(t)$ of the curve

$$\mathbf{r}(t) = \langle \cos(2t), t, \sin(2t) \rangle.$$

Note: for this particular curve, the curvature does **not** depend on the value of t.

3. [10 points] Mark each of the following statements as 'True' of 'False'. If a statement is meaningless, you should mark it as 'False'. (For this question, and for this question only, **you do not have to justify your answers**.)

   (a) **True** / **False**. The dot product is distributive, i.e., $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c}) = \mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$ for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{R}^3$.

   (b) **True** / **False**. The dot product is commutative, i.e., $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$ for all vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$.

   (c) **True** / **False**. The dot product is associative, i.e., $\mathbf{a}\cdot(\mathbf{b}\cdot\mathbf{c}) = (\mathbf{a}\cdot\mathbf{b})\cdot\mathbf{c}$ for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{R}^3$.

   (d) **True** / **False**. The cross product is distributive, i.e., $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$ for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{R}^3$.

   (e) **True** / **False**. The cross product is commutative, i.e., $\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{a}$ for all vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$.

   (f) **True** / **False**. The cross product is associative, i.e., $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{R}^3$.

   (g) **True** / **False**. The equation $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = \mathbf{a}\cdot\mathbf{a}-\mathbf{b}\cdot\mathbf{b}$ holds for all vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$.

   (h) **True** / **False**. The equation $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{b}) = \mathbf{a}\times\mathbf{a}+2\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{b}$ holds for all vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$.

   (i) **True** / **False**. The equation $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = (\mathbf{b}\times\mathbf{c})\cdot\mathbf{a}$ holds for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{R}^3$.

   (j) **True** / **False**. The equation $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = (\mathbf{c}\times\mathbf{b})\cdot\mathbf{a}$ holds for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{R}^3$.

4. [10 points]

(a) Either find the limit and prove that it is what you claim, or prove that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^4}$$

(b) Either find the limit and prove that it is what you claim, or prove that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{\sqrt{x^2+y^2}}$$

(c) Find all points (x, y) where the function f below is continuous. Explain your reasoning.

$$f(x,y) = \begin{cases} \frac{x^2y}{x^2+2y^2}, & \text{if} (x,y) \neq (0,0) \\ -1, & \text{if } (x,y) = (0,0) \end{cases}$$

5. [9 points]

(a) Use a linear approximation of $f(x, y) = e^{xy - y^2}$ at the point $(1, 1)$ to estimate $f(1.02, 1.01)$.

(b) What is the directional derivative of f at the point $(1, 1)$ in the direction of the unit vector $(3/5, 4/5)$?

(c) Find an equation for the tangent plane to the surface $x^4 + y^4 + z^4 = 18$ at the point $(1, 1, 2)$.

6. [6 points] As the in-house mathematician at a landscape gardening firm the following problem arrives on your desk: A manure storage pile is a huge cone of radius 10 feet and height 9 feet. As it is used up, the height decreases by 0.3 feet per day, but due to slippage the radius *increases* by 0.1 feet per day. What is the (initial) rate of manure usage in cubic feet per day?

   Useful fact: the volume of a cone $= \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.