NOTE: In no sense should this collection of problems be construed as representative of the actual exam. These are simply some problems left over from our preparation of the exam or from previous exams which should serve to indicate the general level of expectation.

1. (**Show all work**) Find all points on the surface $2y = 3x^2 + z^3$ at which the tangent plane has normal vector parallel to the line $x = 1 + 3t$, $y = 2 + t$, $z = 3 - 6t$.

2. (**Show all work**) Lines and planes

   (a) Find an equation of the line of intersection of the two planes $x + 2y + 3z = 2$ and $3x - 2y + z = 2$.

   (b) Find the point of intersection of the line $\mathbf{r} = \langle 1 + 2t, 3 + 4t, 5 + 6t \rangle$ and the plane $x + 2y + 3z = 50$.

3. (**Show all work**)

   As a model rocket is launched, it is bumped slightly causing it to have an rather erratic flight path. It is observed that the acceleration of the rocket is given by $\mathbf{a}(t) = \langle -5\cos t, -2\sin t, 0 \rangle$ m/sec^2, with an initial velocity of $\mathbf{v}(0) = \langle 0, 2, 6000/\pi \rangle$ m/sec, and an initial position of $\mathbf{r}(0) = \langle 5, 0, 0 \rangle$. **Note:** There are no other forces acting on the rocket, so don't bring gravity into the picture.

   (a) Determine the velocity $\mathbf{v}(t)$ of the rocket at time t.

   (b) Determine the position $\mathbf{r}(t)$ of the rocket at time t.

   (c) How long does it take for the rocket to reach an altitude of 3000 m?

   (d) A balloon happens to be drifting by following the straight line path $\langle 0, 0, 3000 \rangle + s\langle 1, 2, 0 \rangle$. Does the balloon get hit by the rocket? Why or why not?

4. (**Show all work**) Let $f(x, y) = x^3y^4$.

   (a) Find $\nabla f(1, 1)$.

   (b) Find an equation of the tangent plane to the graph of f at the point $(1, 1, 1)$.

   (c) Find the maximum rate of increase of f at $(1, 1)$, and the direction in which it occurs.

   (d) A unit vector $\mathbf{u}$ makes an angle of $\pi/3$ with $\nabla f(1, 1)$. Find the directional derivative $D_{\mathbf{u}} f(1, 1)$. Hint: you don't really need to know $\mathbf{u}$ to answer this question.

   (e) Determine an equation for the tangent line to the level curve of f at the point $(1, 1)$.

5. **Multiple Choice** Circle the correct response. (No partial credit will be given)

(a) Consider the vector-valued functions $\mathbf{r}(t) = \langle t, t, t \rangle, 1 \leq t \leq e$ and, $\mathbf{s}(t) = \langle e^t, e^t, e^t \rangle, 0 \leq t \leq 1$. Then $\mathbf{r}$ and $\mathbf{s}$ have

A. different velocities and different lengths

B. different speeds and the same length

C. different velocities and the same speed

D. different speeds and different lengths **E**. none of the above

(b) What is the arclength of the piece of the parabola $y = x^2$ from $(0,0)$ to $(2,4)$?

A. $\int_0^4 (1+2t)\,dt$ **B**. $\int_0^4 \sqrt{1+4t^2}\,dt$ **C**. $\int_0^2 \sqrt{t^2+t^4}\,dt$

D. $\int_0^2 \sqrt{1+4t^2}\,dt$ **E**. none of the above

(c) Consider the planes (1) $2(x-2)+3(y-1)-4(z+1)=0$, (2) $2x+3y-2z=11$, and (3) $x+(3/2)y-2z=3$.

A. (1) & (2) are parallel **B**. (2) & (3) are parallel

C. (1) & (3) are parallel **D**. (1), (2) & (3) are parallel

E. none of the above

(d) Let P be the plane given by $x+y+z=3$ and L the line given by $x=t+2$, $y=-t+2$, $z=t+2$. Then

A. L lies on P **B**. L and P are perpendicular

C. L and P are parallel, but L not on P

D. L and P intersect at one point **E**. none of the above

(e) The function $\mathbf{r}(t) = \left\langle \ln t, \dfrac{\sin t}{t}, \dfrac{1}{t} \right\rangle$

A. is continuous at $t = 0$

B. is continuous for all $t > 0$

C. is defined, but not continuous at $t = 0$

D. is discontinuous for all t

E. none of the above

(f) The unit tangent vector of $\mathbf{r}(t) = \langle 1 + t^3, te^{-t}, \sin(2t) \rangle$ at $t = 0$ is

A. $(1/\sqrt{6})\langle 1, 1, 2 \rangle$

B. $(1/\sqrt{5})\langle 0, 1, 2 \rangle$

C. $(1/\sqrt{3})\langle 0, 1, 2 \rangle$

D. $(1/\sqrt{14})\langle 3, 1, 2 \rangle$

E. none of the above

(g) If $\mathbf{u}$ and $\mathbf{v}$ are vectors and the scalar projection of $\mathbf{v}$ onto $\mathbf{u}$ ($\text{comp}_{\mathbf{u}}\mathbf{v}$) is -2, then the angle between $\mathbf{u}$ and $\mathbf{v}$ is

A. acute

B. $\pi/2$

C. obtuse

D. π

E. none of the above