# Math 11 Fall 2004

# Multivariable Calculus

**for Two-Term Advanced Placement First-Year Students**

# Final Exam

Tuesday, December 7, 11:30-2:30
Murdough, Cook Auditorium

Your name (please print): ________________________________________

**Instructions**: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You are allowed to bring one letter-size sheet of paper with any data you want written on it. You must justify all of your answers in this booklet to receive credit, though because this is a multiple choice exam, justifications can be minimal. However, only the answer you mark on the scantron form will be counted.

You have **three hours** to work on all **25** problems. Please do all your work in this exam booklet.

**The Honor Principle requires that you neither give nor receive any aid on this exam.**

Grader's use only

1-25: Multiple Choice: ________ /150

**Total:** ________ /150

(1) Let $\mathbf{F}(x, y, z) = (x + \sin y)\mathbf{i} + (y - \sin(z))\mathbf{j} + z\mathbf{k}$. Then $\operatorname{div} F =$

**a:** $0$;
**b:** $3$;
**c:** $\mathbf{i} + \mathbf{j} + \mathbf{k}$;
**d:** $\cos z\mathbf{i} + 0\mathbf{j} - \cos y\mathbf{k}$.

(2) Let $\mathbf{F}(x, y, z) = (x + \sin y)\mathbf{i} + (y - \sin(z))\mathbf{j} + z\mathbf{k}$. Then $\nabla \times F =$

**a:** $0$;
**b:** $3$;
**c:** $\mathbf{i} + \mathbf{j} + \mathbf{k}$;
**d:** $\cos z\mathbf{i} + 0\mathbf{j} - \cos y\mathbf{k}$.

(3) Apply the Laplace operator to a function $h(x, y, z) = 4xy + e^z$. $\nabla^2 h =$

**a:** 4;
**b:** $e^z\mathbf{k}$;
**c:** $x + y$;
**d:** $e^z$.

(4) Find a function $f(x, y, z)$ such that $f(0, 0, 0) = 1$ and $\nabla f = 2y\mathbf{i} + 2x\mathbf{j} + e^z\mathbf{k}$.

**a:** such $f$ does not exist;
**b:** $f(x, y, z) = 2y^2 + 2x^2 + e^z$;
**c:** $f(x, y, z) = 2xy + e^z + 1$;
**d:** $f(x, y, z) = 2xy + e^z$.

(5) Does there exist a function $f(x, y, z)$ with $\nabla f(x, y, z) = 2xe^{x^2}\mathbf{i} + z\sin(y^2)\mathbf{j} + z^{2004}\mathbf{k}$?

**a:** such $f$ does not exist, since $\nabla^2 f \neq 0$;
**b:** such $f$ does exist;
**c:** such $f$ does not exist, since $\nabla \times \nabla f \neq \mathbf{0}$;
**d:** such $f$ does not exist, since $\operatorname{div}(\nabla f) \neq 0$.

(6) Does there exist a vector field $\mathbf{G}$ with $\mathbf{F}(x, y, z) = \nabla \times \mathbf{G} = e^{yz}\mathbf{i} + \sin(xz^2)\mathbf{j} + z^{2004}\mathbf{k}$?

**a:** such $\mathbf{G}$ does not exist, since $\operatorname{curl}\mathbf{F} \neq \mathbf{0}$;
**b:** such $\mathbf{G}$ does not exist since $\operatorname{div}\mathbf{F} \neq 0$;
**c:** such $\mathbf{G}$ does not exist, since $\mathbf{F}(0, 0, 0) \neq \mathbf{0}$;
**d:** such $\mathbf{G}$ does exist, since $\operatorname{div}(\mathbf{F}) = 0$.

(7) Let $\mathbf{F}(x,y,z) = (x+e^{yz})\mathbf{i} + (y+\sin(x^2z))\mathbf{j} + (z+\cos(x+y))\mathbf{k}$ and let $S$ be the **inwards oriented** sphere $x^2+y^2+z^2=1$. Then $\int\int_S \mathbf{F}\cdot d\mathbf{S} =$

**a:** $4\pi$;
**b:** $\frac{4}{3}\pi$;
**c:** $-4\pi$;
**d:** $1$.

Recall that in the Divergence Theorem the orientation of the surface is **outwards** away from the solid.

(8) Let $F(x,y,z) = x^{2004}\mathbf{i} + \sin(y^2)\mathbf{j} + (x+z)\mathbf{k}$ and let $C$ be a **clockwise oriented** circle with parameterization $\mathbf{r}(t) = \cos(-t)\mathbf{i} + \sin(-t)\mathbf{j} + 0\mathbf{k}$, $t \in [0, 2\pi]$. Then $\int_C \mathbf{F}\cdot d\mathbf{r} =$

**a:** $\pi$;
**b:** $-\pi$;
**c:** $0$;
**d:** $\pi^2$.

(9) Let $C$ be the **clockwise oriented** boundary of the square $\{(x,y) : -1 \leq x \leq 1;\ -1 \leq y \leq 1\}$ in $\mathbb{R}^2$. Then $\int_C x^3dx + (x + \sin y)dy =$

**a:** $\frac{\pi}{3}$;
**b:** 4;
**c:** $-4$;
**d:** 1.

Recall that in Green's Theorem the orientation is counter-clockwise.

(10) Let $\mathbf{F}$ be the force field $\mathbf{F}(x,y,z) = 1\mathbf{i} + 2y\mathbf{j} + 3z^2\mathbf{k}$. The work done by the force $\mathbf{F}$ along a path transporting a particle from the point $(1,1,1)$ to the point $(1,0,0)$ equals:

**a:** $-2$;
**b:** 2;
**c:** 1;
**d:** 0.

(11) The work $\int_C \mathbf{F} \cdot d\mathbf{r}$ done by the force $\mathbf{F}(x, y, z) = x^3\mathbf{i} + (x + y)\mathbf{j} + \sin z\mathbf{k}$ along the closed circular path $C$ given by $\mathbf{r}(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 1\mathbf{k}$, $t \in [0, 2\pi]$, is equal to:

**a:** $-\pi$;
**b:** $9\pi$;
**c:** 0;
**d:** 1.

(12) Let $C$ be a path given by $\mathbf{r}(t) = \langle 1-t, 2-t, 3-t\rangle$, $0 \leq t \leq 1$. Then $\int_C (\mathbf{i}+\mathbf{j}+\mathbf{k}) \cdot d\mathbf{r} =$

**a:** 0
**b:** $-1$;
**c:** 3;
**d:** $-3$.

(13) Let $f(x, y, z) = x + y^2 + z^3$. Then at the point $(0, 1, 1)$ the function $f$ **decreases fastest** in the direction of

**a:** $-\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$;
**b:** $-\mathbf{i} - \mathbf{j} - \mathbf{k}$;
**c:** $\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$;
**d:** $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

(14) For a function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ whose gradient is $\nabla f(x, y) = (x^2 - 1)\mathbf{i} + (\sin y)\mathbf{j}$, the function $f$ has

**a:** zero critical points;
**b:** exactly four critical points;
**c:** exactly two critical points;
**d:** infinitely many critical points.

(15) $\lim_{(x,y)\to(0,0)} \frac{3x^2+y^2}{1-\sqrt{3x^2+y^2+1}}$

**a:** does not exist since the limits along the paths $C_1(t) = (t,t)$ and $C_2(t) = (3t,t)$ are different;

**b:** does not exist since the limits along the paths $C_1(t) = (t,t^2)$ and $C_2(t) = (t,3t^2)$ are different;

**c:** equals $-1$;

**d:** equals $-2$.

(16) $\lim_{(x,y)\to(0,0)} \frac{y}{x^2+y}$

**a:** does not exist since the limits along the paths $C_1(t) = (t,t)$ and $C_2(t) = (3t,t)$ are different;

**b:** does not exist since the limits along the paths $C_1(t) = (t,t^2)$ and $C_2(t) = (t,3t^2)$ are different;

**c:** equals $-1$;

**d:** equals $-2$.

(17) Let $S$ be the **upward oriented** surface that is the part of the cone $z^2 = x^2 + y^2$ located between the planes $z = 1$ and $z = 3$. Let $\mathbf{F}(x, y, z) = \mathbf{k}$ Then $\int\int_S \mathbf{F} \cdot d\mathbf{S} =$

**a:** $8\pi$;
**b:** $-8\pi$;
**c:** $0$;
**d:** $1$.

(18) Let $S$ be the surface that is the part of the cone $z^2 = x^2 + y^2$ located between the planes $z = 1$ and $z = 3$. Then the area of $S$ is equal to

**a:** $16\pi$;
**b:** $12$;
**c:** $8\pi\sqrt{2}$;
**d:** $\pi$.

(19) Find the absolute maximum and the absolute minimum of $f(x, y) = (2x - 1) + y^2$ on the half disk $D = \{(x, y)|y \geq 0, \, x^2 + y^2 \leq 4\}$. The absolute minimum and the absolute maximum are

**a:** $-5$ and 4;
**b:** $-5$ and 3;
**c:** 0 and 2;
**d:** $-3$ and 0.

(20) The function $f(x, y) = \frac{1}{3}x^3 - x + \frac{1}{2}y^2 - y$ has:

**a:** no critical points;
**b:** a local minimum at $(1, 1)$ and a local mimimum at $(-1, 1)$;
**c:** a local minimum at $(1, 1)$ and a saddle point at $(-1, 1)$;
**d:** local maxima both at $(1, 1)$ and at $(-1, 1)$.

(21) Let $W(s,t) = f\big(u(s), v(s,t)\big)$. Assume that $u'(1) = 1$, $\frac{\partial v}{\partial s}(1,3) = 1$, $\frac{\partial v}{\partial t}(1,3) = 4$, $u(1) = 0$, $v(1,3) = 4$, $f_u(0,4) = 1$, and $f_v(0,4) = -2$. Then $\frac{\partial W}{\partial s}(1,3) =$

**a:** $7.5$;
**b:** $0$;
**c:** $-5$;
**d:** $-1$.

(22) The arclength of the curve $\mathbf{r}(t) = \langle t7\sqrt{2}, e^{-7t}, e^{7t}\rangle$, $0 \le t \le 1$ is equal to:

**a:** $12$;
**b:** $e - e^{-1}$;
**c:** $e^7 - e^{-7}$;
**d:** $\pi$.

(23) The area of the triangle with vertices $(2,1,3)$, $(2,1,0)$, $(0,1,3)$ is equal to:

**a:** 2;
**b:** 3;
**c:** 4;
**d:** 5.

(24) The particle is moving with the acceleration $\mathbf{a}(t) = e^t\mathbf{i}$. It has initial velocity $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$ and initial position $\mathbf{r}(0) = \mathbf{i} + \mathbf{k}$. Then $\mathbf{r}(t) =$

**a:** $e^t\mathbf{i} + t\mathbf{j} + \mathbf{k}$;
**b:** $\frac{(e^t)^2}{2}\mathbf{i} + t\mathbf{j} + \mathbf{k}$;
**c:** $(e^t + 1)\mathbf{i} + t\mathbf{j} + \mathbf{k}$;
**d:** $e^t\mathbf{i} + \mathbf{k}$.

(25) Let $E$ be a solid bounded from above by the cone $z = \sqrt{x^2 + y^2}$, bounded from below by the plane $z = 0$, and bounded from the sides by the cylinder $x^2 + y^2 = 1$. Then

$$\int\int\int_E z dV =$$

**a:** $\frac{\pi}{4}$;
**b:** $\frac{\pi}{3}$;
**c:** $\pi$;
**d:** $3 + \pi$.