11/10/15
Barnett

MATH 11 WORKSHEET : Conservative fields

A) Is $\vec{F}(x,y) = (x \cos xy \,,\, y\cos xy + 2y)$ conservative ?

B) Is $\vec{F}(x,y) = (y\cos xy \,,\, x\cos xy + 2y)$ conservative ?

C) For whichever of the above <u>was</u> conservative, find $f(x,y)$ such that $\vec{F} = \vec{\nabla} f$:

$f \xrightarrow{\text{integrate } P \text{ wrt. } x} = \ldots$

[Hint: don't forget $g(y)$!]

take $\frac{\partial}{\partial y}$ of what you got: $f_y = \ldots$

set it equal to Q :

solve for $g'(y) =$

integrate to get $g(y) = \ldots$

Write final answer $f(x,y) = .$

D) For this same $\vec{F}$, find $\int_C \vec{F}\cdot d\vec{r}$ where C is $(t-t^2, t)$, $0 \le t \le 1$:

E) [discuss!] Is $\vec{F}(x,y,z) = (2x + yz^2,\, xz^2,\, 2xyz+1)$ conservative ?

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SOLUTIONS

A) Is $\vec{F}(x,y) = (x\cos xy\,,\ y\cos xy + 2y)$ conservative ?

$\frac{\partial P}{\partial y} = -x^2 \sin xy \neq \frac{\partial Q}{\partial x} = -y^2 \sin xy \Rightarrow$ no.

B) Is $\vec{F}(x,y) = (y\cos xy\,,\ x\cos xy + 2y)$ conservative ?

$\frac{\partial P}{\partial y} = \cos xy - xy\sin xy \qquad \frac{\partial Q}{\partial x} = \cos xy - xy \sin xy$

Equal, and domain is all of $\mathbb{R}^2$ $\Rightarrow$ yes.

C) For whichever of the above was conservative, find $f(x,y)$ such that $\vec{F} = \vec{\nabla} f$:

B)

$f \overset{\text{integrate } P \text{ wrt. } x}{=} \int y\cos xy\, dx = \sin xy + g(y)$ [Hint: don't forget $g(y)$!

take $\frac{\partial}{\partial y}$ of what you got : $f_y = \ldots\ x\cos xy + g'(y)$

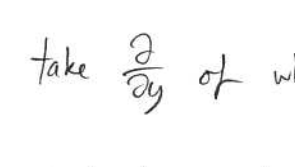
set it equal to Q : $x\cos xy + g'(y) = Q = x\cos xy + 2y$

solve for $g'(y) = 2y$

integrate to get $g(y) = \ldots\ y^2 + c$

Write final answer $f(x,y) = \ldots\ \sin xy + y^2 + c$

D) For this same $\vec{F}$, find $\int_C \vec{F}\cdot d\vec{r}$ where C is $(t - t^2, t)$, $0 \le t \le 1$:



$\xleftrightarrow{\text{by FTLI}} = f(0,1) - f(0,0) = 1^2 - 0^2 = 1$

E) [discuss!] Is $\vec{F}(x,y,z) = (\overset{P}{2x + yz^2}, \overset{Q}{xz^2}, \overset{R}{2xyz + 1})$ conservative ?

yes, since $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$, $\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$, & $\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}$ 3 conditions, domain = $\mathbb{R}^3$