Math 74/114, Spring 2017

Homework set 7, due Wed May 24

This homework set is due on Wednesday May 24, at the start of class. Discussion of the problems is permitted, and even recommended. But you should write up and hand in your own solutions.

1. Do Hatcher's Exercises 4, 9ab, 12, 34 from section 2.2, p.155-158.

2. Let M_ϕ be the 3-dimensional manifold obtained by attaching the boundary of a solid torus $A = D^2 \times S^1$ to the boundary of another solid torus $B = D^2 \times S^1$ via an attaching map

$$\phi : \partial A = S^1 \times S^1 \to \partial B = S^1 \times S^1$$

The attaching map ϕ is defined as follows. Identify S^1 with the set of unimodular complex numbers $S^1 = \{z \in \mathbb{C} : |z| = 1\}$. Then

$$\phi(z, w) = (z^a w^b, z^c w^d) \qquad z, w \in S^1 \subset \mathbb{C}$$

where a, b, c, d are integers. We assume that $ad - bc = 1$. Then it is easy to verify that ϕ is a homeomorphism with inverse $(z, w) \mapsto (z^d w^{-b}, z^{-c} w^a)$.

   (a) Calculate the homology groups $H_\bullet(M_\phi)$ of the manifold M_ϕ.

   (b) Identify all maps ϕ so that M_ϕ has the same homology as S^3.

   (c) Identify at least one map ϕ so that M_ϕ is homeomorphic to S^3.