Math 112
Introduction to Riemannian Geometry
Spring 2006
Assignment 5
Due May 30, 2006

Chp. 5 (do Carmo) 2, 3 & 5
Chp. 8 (do Carmo): 5

1. Let $\gamma : [a,b] \to (M,g)$ be a geodesic and let $\mathcal{V}_\gamma$ be the vector space of piecewise smooth vector fields along γ. Recall that the index form along γ is the bilinear form $I : \mathcal{V}_\gamma \times \mathcal{V}_\gamma \to \mathbb{R}$ given by

$$I(V,W) \equiv \int_a^b \{\langle V', \mathcal{W}' \rangle - \langle R(\gamma', V)\gamma', W \rangle\} dt.$$

Now let $\mathcal{V}_\gamma^0 = \{V \in \mathcal{V}_\gamma : V(a) = V(b) = 0\}$. Show that a vector field J along γ is a Jacobi field if and only if $I(J,V) = 0$ for every $V \in \mathcal{V}_\gamma^0$.