Math 112
Introduction to Riemannian Geometry
Spring 2006
Assignment 2
Due April 20, 2006

Problems from do Carmo
Chp. 1: 1, 2, & 4
Chp. 2: 1, 3, 4 & 5

One-parameter Subgroups

1. A **one-parameter subgroup** of a Lie group G is a smooth homomorphism $\phi : (\mathbb{R}, +) \to G$. Prove that there is a one-to-one correspondence between the one-parameter subgroups of G and the left-invariant vector fields on G.

2. For each $X \in \mathfrak{g}$ let ϕ_X denote the unique one-parameter subgroup of G such that $\phi_X'(0) = X_e$. We can then define the exponential map $\exp : \mathfrak{g} \to G$ given by

$$X \mapsto \phi_X(1).$$

   We sometimes denote this map by $e^X = \exp(X)$. Show that for each $X \in \mathfrak{g}$ the map $\gamma(t) = e^{tX}$ is a group homomorphism with $\gamma'(0) = X$.

Left-Invariant Metrics

1. Recall that an automorphism of a group G is an isomorphism $\alpha : G \to G$. Let G be a Lie group equipped with a left-invariant metric $\langle \cdot, \cdot \rangle$ and let $\alpha : G \to G$ be a smooth automorphism of G.

   (a) Show that $(\cdot, \cdot) \equiv \alpha^* \langle \cdot, \cdot \rangle$is a left-invariant metric.

   (b) Show that $(G, \langle \cdot, \cdot \rangle)$ and $(G, (\cdot, \cdot))$ are isometric.