Mathematics 111

Spring 2007

Homework 2

1. Let A be a ring with identity and consider the short exact sequence of A-modules:

$$0 \longrightarrow M' \xrightarrow{\varphi} M \xrightarrow{\psi} M'' \longrightarrow 0$$

   (a) Show that if M' and M'' are finitely generated, so is M.

   (b) Show that if M is finitely generated, so is M''.

   (c) Show by example that if M is finitely generated, M' need not be.

2. Problem 5, page 166 of Lang: Let A be an additive subgroup of $\mathbb{R}^n$ (i.e. a $\mathbb{Z}$-module). Suppose that for any bounded subset B of $\mathbb{R}^n$, $A \cap B$ is finite. Show that A is a free $\mathbb{Z}$-module of rank $m \leq n$.

   Following Artin and Whaple's original proof, Lang gives a detailed hint. Make sure you work out the base case carefully and the inductive step will be easier.