

Mathematics 111
Spring 2007
Homework 1

Prove the following theorem (Proposition 2.2, Chapter 3 of Lang) which is analagous to the “contravariant” one proved in class.

Theorem: Let A be a ring with identity. The sequence of left A -modules

$$0 \longrightarrow N' \xrightarrow{\varphi} N \xrightarrow{\psi} N''$$

is exact if and only if for all left A -modules M , the sequence

$$0 \longrightarrow \text{Hom}_A(M, N') \xrightarrow{\varphi_*} \text{Hom}_A(M, N) \xrightarrow{\psi_*} \text{Hom}_A(M, N'')$$

is exact.

Hint: For the converse, a single choice of M can work for all parts, but you still need to sweat the details.