

---

Note to grader: please accept some rounding errors. E.g. using a “close enough” value in the table is fine.

### Instructions

- Type your answers and paste images directly into this document.
- Or add additional space, print this out, and fill it in by hand.
- You will probably need to use a **calculator** for this homework.
- Print out and hand in homework in class on Tuesday.
- You may collaborate on the homework but you must write it up yourselves.

### Hypothesis Testing For Difference Between Two Means

#### Question 1 - 15 points

Suppose we have two normally distributed populations 1 and 2, with the same unknown variance. You take two samples from them.

Sample 1	-	sample mean = 18,	sample SD = 2,	size n = 10
Sample 2	-	sample mean = 15,	sample SD = 3,	size n = 10

“Sample SD” is the estimator of the standard deviation, which is calculated using the data in the sample.

Perform a one-tailed hypothesis test at the 3% level of significance, on whether these two populations have the same population mean, or if population 1 has a larger mean.

**Answers :**

**Null hypothesis : difference between population mean = 0 (can write  $\mu_1 - \mu_2 = 0$  too).**  
**Alternative : population mean 1 greater than 2. (only one-tailed tests accepted)**

$$SE = \sqrt{(4+9)/10} = 1.14.$$

$$\text{Degrees of freedom} = 2(10-1) = 18.$$

$$t\text{-statistic} = (18-15)/1.14 = 2.63.$$

$$P(\text{difference} > 18 - 15 = 3) = P(T > 2.63) < P(T > 2.55) = 0.01 < 0.03.$$

**Null rejected at 3% level of significance. Pop. 1 probably has a larger mean.**

## Question 2 - 15 points

Suppose we have two populations.

Population 1 - population variance = 15  
Population 2 - population variance = 20

Sample 1 - sample mean = 97, size n = 20  
Sample 2 - sample mean = 90, size n = 30

Perform a two-tailed hypothesis test at the 10% level of significance, on the hypothesis that population 1 and population 2 has the same population mean.

Answers:

Null hypothesis : pop. mean same (or diff. in pop mean = 0, or  $\mu_1 - \mu_2 = 0$ ).

Alternative hypothesis : pop mean difference (or diff. in pop mean not 0, or  $\mu_1 - \mu_2 \neq 0$ ).

$$SE = \sqrt{15/20 + 20/30} = 1.190$$

$$z\text{-statistic} = (97 - 90)/1.190 = 5.882$$

$$P(Z \geq 5.882) < 0.001 \text{ for sure.}$$

So, we reject the null at 10% level of significance. The two population probably do not have the same population mean.

Grader: they do not have to use the two-tailed p-value of  $2 * P(Z \geq 5.882)$  for this course. But they should also be given full credit for doing so. Also, it is fine to call  $P(Z \geq 5.882)$  the p-value for this course.