Math 10 – Homework 4

Lots of decimals here. Please do not penalize for rounding errors, unless it is too severe. E.g. rounding 0.072 to 0.1 should get at least 1 point off. E.g. 0.3679 to 0.368 is fine. Use your judgement, I will fix any problems that come up.

Note to grader 1

We asked the students to stick to two-tailed tests if they are unsure. Some might end up using one-tailed tests. Both are fine.

Note to grader 2

One-tailed and Two-tailed p-values are in the textbook but my (Tommy's) section is not using them. We are sticking to one-tailed p-values and checking the condition p-value > alpha (for 1 tailed test) and p-value > alpha/2 (for 2 tailed test). Alpha is the significance level. This is mathematically equivalent.

Instructions

- Type your answers and paste images directly into this document.
- Or add additional space, print this out, and fill it in by hand.
- You will probably need to use a calculator for this homework.
- Print out and hand in homework in class on Tuesday.
- You may collaborate on the homework but you must write it up yourselves.

Question 1

(10 pts) An exit poll of 1000 randomly selected voters found that 515 favored candidate A. Is the race too close to call based on just this random sample? Answer this question by performing an appropriate test of hypothesis at 1% level of significance. *(Hint: use the Normal approximation to the Binomial distribution, Luke)*

Null hypothesis: 0.50 (too close to call)
Alternative hypothesis: not equal to 0.50.

Suppose the null hypothesis is true. Then, applying the Normal approximation to the Binomial distribution, we a get sampling distribution that is approximately Normal with mean 0.50 and standard error SE is sqrt(0.5*0.5/1000) = 0.0158. (please do not penalize rounding errors)

0.515 = 515/1000 is the sample proportion. The p-vale is P(X >= 0.515) = P((X – mean)/SE >= (0.515 – 0.50)/0.0158) = P(Z >= 0.94936) = 1 – 0.8289 = 0.1711.

So, the p-value here is around 0.1711 (please do not penalize rounding errors).

We cannot reject the null at 1% significance level. The race is *probably* too close to call.

Question 2

(10pts) The time needed for college students to complete a certain maze follows a normal distribution with a mean of 45 seconds. To see if the mean time μ (in seconds) is changed by vigorous exercise, we have a group of nine college students exercise vigorously for 30 minutes and then complete the maze. The sample mean and an estimator of the standard deviation, calculated from the collected data, is 49.2 seconds and 3.5 seconds respectively. Use these data to perform an appropriate test of hypothesis at 5% level of significance

Null hypothesis: mean = 45
Alternative hypothesis: mean not 45.

Population is Normally distributed, variance unknown, so do t-test.

Df = 9 – 1 = 8. SE = 3.5/sqrt(9) = 1.16667.

P(sample mean >= 49.2)
= P((sample mean – null hypothesis)/1.1667 >= (49.2 – 45)/1.1667)
= basically 3.6

P(t-value >= 3.6) <= P(t-value >= 3.36) = 0.005.

The (one-tailed) p-value is less 0.005 which is less than 0.025, and so we reject the null hypothesis at the 5% level of significance.

The mean time is *probably* changed by vigorous exercise.

Question3

(10 pts) A research team is interested in the difference between serum uric acid levels in patients with and without Down's syndrome.

In a large hospital for the treatment of the mentally retarded, a sample of 15 individuals with Down's syndrome yielded a mean of 4.5 (mg per 100ml). In a general hospital a sample of 12 normal individuals of the same age and sex were found to have a mean value 3.4 (mg per 100ml).

If it is reasonable to assume that the two populations of values are normally distributed with variances equal to 1 and 1.5 respectively, find the 95 percent confidence interval for difference of means

Note: the sampling distribution for the difference between two means is normal when the population variances are known.

Sample difference between means = 4.5 – 3.4 = 1.1 * 3.4 – 4.5 is fine too
SE when variances are known: sqrt(1/15 + 1.5/12) = 0.4378

95% C.I. is [sample diff. in means -+ z-value * SE]
= [1.1 - 1.96*0.4378, 1.1 + 1.96*0.4378] = [0.241912,1.958].

Question 4
(10 pts) Construct a 90% confidence interval for the difference of population means, for two normally distributed population with the same but unknown variance. Assume that the two samples, each taken from a different population, are independent.

Sample 1: sample mean = 49.37, sample SD = 4.89, n = 16
Sample 2: sample mean = 52.13, sample SD = 5.38, n = 16

"Sample SD" is the estimator of the standard deviation, calculated using the sample.

Please forgive rounding errors in this question.

Sample diff. in mean = 49.37 – 52.13 = -2.76 (52.13 – 49.37 is fine too).

MSE = (4.89^2 + 5.38^2)/2 = 26.4282.

Estimated SE = sqrt(2MSE/n) = sqrt(2*26.4282/16) = around 1.8175.

Same unknown variance and populations normal => t-distribution.

Degrees of freedom = (n – 1) + (n – 1) = 30.
t-value is 1.70.

90% CI = [-2.76 - 1.70*1.8175, -2.76 + 1.70*1.8175] = [-5.84975, 0.32975].

* It is fine to use the 52.13 – 49.37 version.