Name: KEY

Math 10 Midterm 1
April 21, 2010

Read all instructions carefully. No calculators are allowed. You may leave answers unevaluated; e.g., $6(\frac{1}{2})^5$. This is a closed book exam and no notes are allowed; the table of areas under the normal curve from the book is provided on the back of this page. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have two hours and you should attempt all 11 problems. There will be partial credit, so show all work.

FERPA RELEASE: Because of privacy concerns, we are not allowed to return your graded exams in lecture without your permission. If you wish us to return your exam in lecture, please sign on the line indicated below. Otherwise, you will have to pick your exam up in your instructor's office after the exams have been returned in lecture.

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(1) (12 pts) A die is rolled 9 times. Let event A be that the first five rolls contain exactly two 6s, and event B be that the last four rolls contain exactly one 5.

(a) Find the probability of A *and* B.

$$\text{prob}(A) = \binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{10 \cdot 5^3}{6^5}$$

$$\text{prob}(B) = \binom{4}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 = \frac{4 \cdot 5^3}{6^4}$$

Content of first 5 rolls does not influence probabilities of last 4 rolls — A and B are independent.

$$\text{prob}(A \text{ and } B) = \text{prob } A \cdot \text{prob } B = \frac{40 \cdot 5^6}{6^9}$$

(b) Find the probability of A *or* B.

Inclusion - exclusion

$$\text{prob}(A \text{ or } B) = \text{prob}(A) + \text{prob}(B) - \text{prob}(A \text{ and } B)$$

$$= \frac{10 \cdot 5^3}{6^5} + \frac{4 \cdot 5^3}{6^4} - \frac{40 \cdot 5^6}{6^9}$$

(2) (8 pts) Define the following events for a standard deck of 52 cards.

A first card is a heart
B first card is the king of spades
C first card is a face card
D first card is a five

(a) Which pairs of events are independent, if any?

A/C

A/D

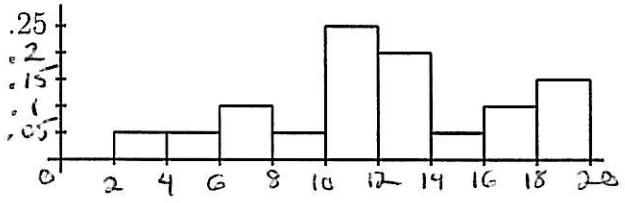
(b) Which pairs of events are mutually exclusive, if any?

A/B

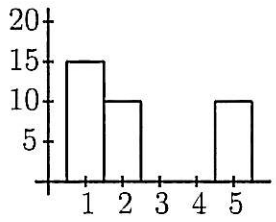
B/D

C/D

(3) (6 pts) A certain data set has range 2-20, median 12, and lower and upper quartiles 10 and 16. Given that the marks on the axes of the following percentage histogram are to scale (within each axis; the x-scale need not equal the y-scale), label all the axis markings.



(4) (6 pts) Given the following frequency histogram for a data set, find the average. You may assume all values in the set are whole numbers.



data points: $15 + 10 + 10 = 35$

sum of data points: $15 \cdot 1 + 10 \cdot 2 + 10 \cdot 5$

$= 85$

mean: $\frac{85}{35} = \frac{17}{7}$

(5) (9 pts) Suppose you have a data set with mean 2 and standard deviation 1.3. For each of the following pairs of operations on the data, say which operation has a greater effect on (i) the mean, and (ii) the standard deviation (that is, which changes the value by a larger amount). If they do the same thing, say "no difference".

(a) Adding 2 to each entry in the data set versus multiplying each entry by 2.

(i) mean + 2 = 4
mean · 2 = 4
no difference

(ii) SD unaffected by +2
SD · 2 = 2.6
mult. by 2

(b) Subtracting 4 from each entry in the data set versus multiplying each entry by -1.

(i) mean − 4 = −2
mean · (−1) = −2
no difference

(ii) SD unaffected by −4
SD · |−1| = SD
no difference

(c) Multiplying each entry in the data set by -3 versus adding 4 to each entry.

(i) mean · (−3) = −6 [down by 8]
mean + 4 = 6
mult. by −3

(ii) SD · |−3| = 3.9
SD unaffected by +4
mult by −3

(6) (6 pts) If for a certain data set, outcome value 2 is 0.5 in standard units, and outcome value 5 is 1.5 in standard units, what are the mean and standard deviation of the data set?

$$\frac{2-\bar{x}}{SD} = \frac{1}{2} \qquad 4 - 2\bar{x} = SD$$

$$\frac{5-\bar{x}}{SD} = \frac{3}{2} \qquad 10 - 2\bar{x} = 3SD$$

many routes to solution; here are two

$10 - 2\bar{x} = 3SD$

$-\ [4 - 2\bar{x} = SD]$

$6 = 2SD$

$\boxed{SD = 3}$

$4 - 2\bar{x} = 3$

$1 = 2\bar{x}$

$\boxed{\bar{x} = \frac{1}{2}}$

$10 - 2\bar{x} = 3(4 - 2\bar{x})$

$10 - 2\bar{x} = 12 - 6\bar{x}$

$4\bar{x} = 2$

$\boxed{\bar{x} = \frac{1}{2}}$

$4 - 2 \cdot \frac{1}{2} = SD$

$\boxed{SD = 3}$

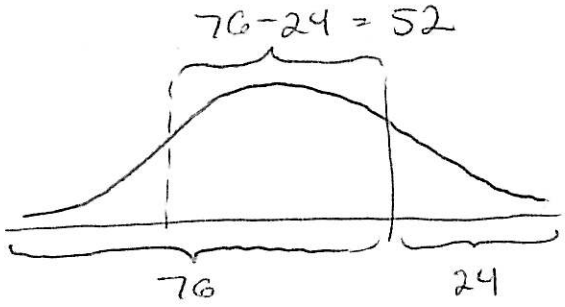
Shortcut: $2 \mapsto \frac{1}{2}$, $5 \mapsto \frac{3}{2}$ > 1 SD apart; $SD = 5 - 2 = 3$

$\frac{1}{2} SD = 1.5$

$2 - 1.5 = \bar{x} = .5$

(7) (12 pts) A normally distributed data set has mean 12 and standard deviation 4.

(a) What is the 76th percentile (approximately)?



area = 52

$z = 0.7$ (approx)

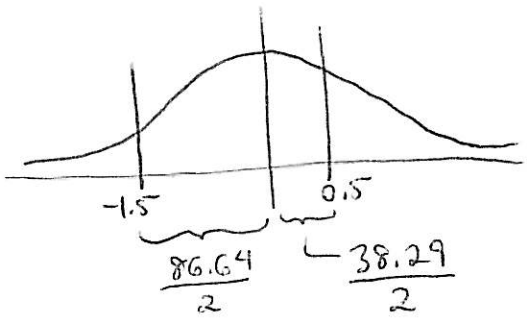
0.7 in orginal units: $0.7 \cdot 4 + 12 = 2.8 + 12$

$= 14.8$

(b) What percentage of outcomes were between 6 and 14?

$$\frac{6-12}{4} = \frac{-6}{4} = -1.5$$

$$\frac{14-12}{4} = \frac{2}{4} = 0.5$$



area -1.5 to 1.5 86.64

-0.5 to 0.5 38.29

total area (% outcomes): $43.32 + 19.15 = 62.47$

(8) (12 pts) 8 green marbles and 3 blue marbles are in a box.

(a) If 5 draws are made without replacement, what is the probability that the first and last draws will be blue and the rest will be green?

B G G G B

$$\frac{3}{11}\cdot\frac{8}{10}\cdot\frac{7}{9}\cdot\frac{6}{8}\cdot\frac{2}{7}$$

(b) Repeat part (a) but now assume draws are made with replacement.

B G G G B

$$\frac{3}{11}\cdot\frac{8}{11}\cdot\frac{8}{11}\cdot\frac{8}{11}\cdot\frac{3}{11}=\frac{3^2 8^3}{11^5}$$

(9) (14 pts) 1 blue, 2 red, and 3 green marbles are in a box. 3 are drawn with replacement. Let event A be "at least two drawn are blue", event B be "at least two drawn are red", and event C be "at least two drawn are green".

(a) Find the probabilities of each individual event.

each is the sum of the probabilities of mutually exclusive events "exactly two" and "exactly three"

$$\text{prob}(A) = \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^3 = \frac{16}{6^3} = \frac{2}{27}$$

$$\text{prob}(B) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^3 = \frac{7}{3^3} = \frac{7}{27}$$

$$\text{prob}(C) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3 = \frac{4}{2^3} = \frac{1}{2}$$

(b) What is prob(A given B)?

If a draw of three balls has at least two red, it cannot also have at least two blue.

$$\text{prob}(A \text{ given } B) = 0$$

(c) What is prob(A or B or C)?

as in (b) all three events are mutually exclusive, so

$$\text{prob}(A \text{ or } B \text{ or } C) = \text{prob}\,A + \text{prob}\,B + \text{prob}\,C$$
$$= \frac{2}{27} + \frac{7}{27} + \frac{1}{2} = \frac{5}{6}$$

(10) (6 pts) Suppose you know $\binom{990}{30}$; call it x.

(a) What is $\binom{990}{960}$ in terms of x?

$$x = \binom{990}{30} = \frac{990!}{960!\,30!}$$

$$\binom{990}{960} = \frac{990!}{30!\,960!} \quad (= x)$$

(b) What is $\binom{990}{29}$ in terms of x?

$$\binom{990}{29} = \frac{990!}{29!\,961!}$$

$$= \frac{30}{30} \cdot \frac{990!}{29!\,960!} \cdot \frac{1}{961}$$

$$= \frac{30}{961} \cdot \frac{990!}{30!\,960!}$$

$$= \frac{30}{961} \cdot x$$

(11) (9 pts) Suppose you have a data set with mean 4.5 and standard deviation 1. What will happen to the mean and standard deviation if you augment the data set by adding the following values to it? You do not need numerical values, just an explanation.

(a) 4, 4, 4, 5, 5, 5

mean of these points is 4.5 so overall mean does not change

these points lie within one unit of the mean, so SD will decrease.

(b) 2, 2, 2, 7, 7, 7

as above, mean does not change

these points lie beyond one unit of the mean, so SD will increase.

(c) 12, 12, 12

mean of these points is 12 so overall mean will increase

these points count as outliers to the orginal set so the SD will also increase