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Consider the experiment of throwing two dice and observing the ordered outcomes. Then consider the events

- Event A, the sum of the two outcomes is even.
- Event B, the sum of the two outcomes is 6.

You can use this table to help yourself with the computations:

$(1,1)$	$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(1,6)$
$(2,1)$	$(2,2)$	$(2,3)$	$(2,4)$	$(2,5)$	$(2,6)$
$(3,1)$	$(3,2)$	$(3,3)$	$(3,4)$	$(3,5)$	$(3,6)$
$(4,1)$	$(4,2)$	$(4,3)$	$(4,4)$	$(4,5)$	$(4,6)$
$(5,1)$	$(5,2)$	$(5,3)$	$(5,4)$	$(5,5)$	$(5,6)$
$(6,1)$	$(6,2)$	$(6,3)$	$(6,4)$	$(6,5)$	$(6,6)$

1. What is the probability of event B occurring?

2. What is the probability of event B occurring if you know that event A occurred?

3. What is the probability of event B occurring if you know that event A did not occur?

The probability of an event B happening if an event A happened, is called the conditional probability of B given that A has occurred and is denoted by $P(B|A)$. In general this probability is

$$P(B|A) = \frac{n(A \textit{ and } B)}{n(A)}$$

4. If a father of two told you that at least of one his children is a male, what are the odds that the other one is a female?

5. What is the probability that rolling three dice the sum is 11 if you know that the product of the first two outcomes is 4?

The previous formula can be written as

$$P(B|A) = \frac{n(A \textit{ and } B)}{n(A)} = \frac{n(A \textit{ and } B)/n(S)}{n(A)/n(S)} = \frac{P(A \textit{ and } B)}{P(A)}$$

6. In a town, 70% of the men are employed. The probability that a man will commit a crime is 10%, and the probability that a man is employed, given he will commit a crime, is 5%. A man is selected by chance and is employed. What is the probability that he will commit a crime?

From the previous, the following rule can be deduced.

Multiplication rule If A and B are events in some sample space, then

$$P(A \textit{ and } B) = P(A) \cdot P(B|A)$$

7. What is the probability that spinning the roulette (the one with 38 numbers) you get for three times in a row a number in between 1 and 6, such that the product of the first two numbers is 4 and the sum of the three numbers is 11?

Bayes Rule

The previous formula can be written in a symmetric way as

$$P(B) \cdot P(A|B) = P(A \text{ and } B) = P(A) \cdot P(B|A)$$

This leads to the famous Bayes theorem

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$

A typical application of Bayes theorem is in medical tests. Suppose you perform an HIV test on a patient, for screening purposes, and the following is true about the test.

- If a tested patient has HIV, the test returns a positive result 99% of the time
- If a tested patient does not have HIV, the test returns a positive result 1% of the time

Suppose also that on average 0.1% of the patients have HIV. Call A the event of having HIV and B the event of being tested positive.

8. Write down and compute the following:
   - $P(B|A) =$ ______________________________
   - $P(B|\bar{A}) =$ ______________________________
   - $P(A) =$ ______________________________
   - $P(B) =$ ______________________________

9. If the patient is tested positive, what are the odds that the patient has HIV?

10. If the patient is tested negative, what are the odds that the patient does not have HIV?

11. Would you suggest the test for sale? If not, what parameters should have been different in the medical test?