

NAME: _____

MATH 10 MIDTERM 2

May 12, 2008

INSTRUCTIONS: This is a closed book, closed notes exam. You are not allowed to provide or receive help from any outside source during the exam.

- No calculators are allowed.
- You must show your work to receive full credit.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

Signature

Question	Points	Score
1	3	
2	3	
3	3	
4	3	
5	15	
6	6	
7	10	
8	3	
9	5	
10	5	
11	3	
12	18	
13	9	
14	6	
15	8	
Total:	100	

For a normal population with mean μ and standard deviation σ we expect

Between	Percentage of Data
$\mu - \sigma$ and $\mu + \sigma$	68.26%
$\mu - 2\sigma$ and $\mu + 2\sigma$	95.44%
$\mu - 3\sigma$ and $\mu + 3\sigma$	99.74%

1. [3 points] A company has 6 employees, including the owner. The sum of the salaries of the 5 non-owners is less than the salary of the owner. Assuming that no two salaries are the same, how many employees earn less than the mean salary for the company and why? How many employees earn less than the median salary for the company?

2. [3 points] Give a list of 4 different integers between 1 and 10 (inclusive) such that the standard deviation is minimal.

3. [3 points] Give a list of 4 different integers between 1 and 10 (inclusive) such that the standard deviation is maximal.

4. [3 points] Compute the correlation between the following two lists:

$$\mathbf{x} = [16 \ 31416];$$

$$\mathbf{y} = [271828 \ 9];$$

5. Every Tuesday evening, Rarito likes to roll a 6-sided die 4 times and he records the outcomes on a piece of paper.

(a) [3 points] What is the number of ordered outcomes?

(b) [3 points] What is the number of ordered outcomes in which no number is repeated?

(c) [3 points] What is the probability that Rarito does not write down the same number twice?

(d) [3 points] Knowing that at least a number was repeated, what is the probability that Rarito first writes a 2, then a 3, then again a 2, and finally a 4?

(e) [3 points] Knowing that no number was repeated, what is the probability that neither a 1 nor a 2 were rolled?

6. Rarito rolls one of his 6-sided dice exactly 6 times.

(a) [3 points] What are the odds that he rolls a number larger than or equal to 5 exactly three times?

(b) [3 points] What are the odds that he rolls at least twice a number larger than or equal to 5?

7. After dancing salsa all night for the Cinco de Mayo, Rarito comes back home and he starts to roll one of his 6-sided dice 72 times and he counts the number x of times he obtains a 5 or a 6.

(a) [3 points] What is the expected value of x ?

(b) [3 points] What is the standard deviation of x ?

(c) [4 points] What are, approximately, the odds that x is at least 32?

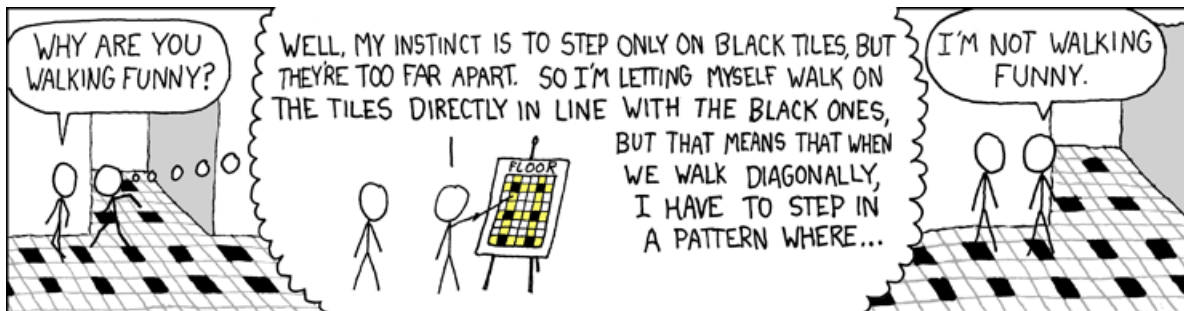
8. [3 points] Check which of the following statements holds for a normal distribution.

- (a) The interquartile range is larger than the standard deviation.
- (b) The standard deviation is larger than the interquartile range.
- (c) The interquartile range is larger than two standard deviations.
- (d) a and c.
- (e) b and c.

9. [5 points] Is it more likely to get between 40 and 60 heads (inclusive) in tossing 100 fair coins once, or between 400 and 600 heads (inclusive) in tossing 1000 fair coins once? Explain.

10. [5 points] As a Christmas present, Freaky bought Rarito a 4-sided fair die. After spending the whole night playing with his new die, Rarito wonders if it is more likely that the sum of 70 outcomes of the 4-sided die falls in between 150 and 200 (inclusive) or that the sum of 50 outcomes of the regular 6-sided die falls in between 150 and 200 (inclusive). Can you help him?

11. [3 points] A normal random variable x has mean μ and standard deviation σ . Overall 6 values are sampled and the sample mean \bar{x} and the sample variance s^2 are computed. Do you expect the value found for s^2 to be the same as σ^2 , larger, or smaller? Explain.



12. Freaky sometimes likes to play with Rarito's dice. He picks two fair 6-sided dice, rolls them, and records the outcome X of the first die, the outcome Y of the second die, the sum $P = X + Y$, the sum $Q = X + Y + P$, the product $R = X \times Y$ and the product $S = X \times P$. Compute the following (you can use the fact that expectation and variance for a single roll are respectively $7/2$ and $35/12$)

(a) [3 points] $E(P) =$

(b) [3 points] $V(P) =$

(c) [3 points] $E(Q) =$

(d) [3 points] $V(Q) =$

(e) [3 points] $E(R) =$

(f) [3 points] $E(S) =$

13. Rarito and Freaky occasionally play dice together. Two fair 6-sided dice are rolled and if the sum is 9 Rarito has to pay Freaky 7 dollars, while in the other case Freaky has to pay Rarito 2 dollars. Say x is the amount of money Rarito wins.

(a) [3 points] What is the probability Rarito loses money?

(b) [3 points] What is the expected value of the random variable x ?

(c) [3 points] What is the variance of the random variable x ?

14. [6 points] Suppose the previous game gets played 32 times. What are the odds Rarito loses money now?



15. Assume that x is a random variable with mean $\mu = 60$ and standard deviation $\sigma = 5$. The following six samples are obtained:

57 56 67 65 63 64

and the sample mean \bar{x} is computed.

(a) [3 points] Compute the z -value for \bar{x} .

(b) [3 points] Compute the t -value for \bar{x} .

(c) [2 points] Which one is larger? Is it always the case? Explain.

