LECTURE OUTLINE
Probability

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Math 10

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The Abstraction: Events

$$U = \{\text{All possible outcomes of an experiment}\}$$

An event is any subset of U, denoted by

$$A \subset U.$$

The *union* of two events A and B is denoted as

$$A \bigcup B.$$

The *complement* of the event A is denoted as

$$\tilde{}A.$$

The Abstraction: Probability

Using the notation from the previous slide, we have

1. $P(U) = 1$
2. $P(A) \geq 0.$
3. If A and B have no outcomes in common then $P(A \bigcup B) = P(A) + P(B)$.

The Equally Likely Case

If there are $|U|$ possible outcomes and each is *equally likely* (usually due to *symmetry*), then the probability of an event E that is collection of $|E|$ outcomes satisfies

$$P(E) = \frac{|E|}{|U|}.$$

This reduces finding probabilities to *counting*.

Equally Likely: Example 1

Coin Flips: Flip a fair coin 5 times.

Compute

$$P(\text{ At least 4 of the flips are Heads}),$$

$$P(\text{ At least 4 of the flips are Tails}),$$

and

$$P(\text{ At least 4 of the flips are the same type}).$$

Equally Likely: Example 1 (continued)

Coin Flips: Flip a fair coin 5 times.

Compute

$$P(\text{ Not all the flips are the same}).$$

A "Lemma"

One way to compute the previous example is to observe

$$1 = P(A \bigcup \tilde{} A) = P(A) + P(\tilde{} A),$$

so

$$P(\tilde{} A) = 1 - P(A).$$

Equally Likely: Example 1 (continued)

Coin Flips: Flip a fair coin 5 times.

Compute

$$P(\text{Exactly 3 Heads}).$$

A Key Counting Fact

The number of ways to choose k distinct objects from a collection of n distinct objects is denoted as $\binom{n}{k}$. We read this expression as "n choose k".

One way to compute the previous example is to apply the following formula for "n choose k":

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} = \frac{n \cdots (n-k+1)}{k \cdots 1}.$$

Equally Likely: Example 2

Poker Hand: Deal out a 5 card poker hand from a "randomly" mixed up standard deck (no jokers). Check to see that the higher ranked hands have a lower probability associated to them. Hands are ranked as follows.

1. Royal Flush
2. Straight Flush
3. Four of a kind
4. Full House
5. Flush
6. Straight
7. Three of a kind
8. Two Pairs
9. Pair
10. High Card

The Monty Hall Problem

Student, "....it was called the "Monty Hall Problem" or something like that, after the host of "let's make a deal." It goes something like this: As you probably know, on the show there are three doors. Two contain a goat and one, a sports car. This is so that no matter what door the contestant chooses, monty can then open the other door that he knows contains a Goat and ask the person if he would like to switch his prize for the one behind the remaining door that he has not chosen. The prof claims that the person should ALWAYS switch his door because it increases his chances for winning..."

The Monty Hall Problem As Asked by Marilyn

Ask Marilyn by Marilyn vos Savant is a column in Parade Magazine. In the Ask Marilyn column of September 9, 1990, February 17, 1991, and July 7, 1991, Marilyn introduced this problem as: a gameshow in which the contestant is given a choice of one of three doors, behind one of which is a prize. A reader asked if, after the contestant chooses a door, the host opens a different door, revealing no prize, and offers the contestant the opportunity to switch doors, whether the contestant should switch or not.

Quotes By Monty Hall Himself

"If the host is required to open a door all the time and offer you a switch, then you should take the switch," he said. "But if he has the choice whether to allow a switch or not, beware. Caveat emptor. It all depends on his mood."

(I guess after a certain contestant guessed the wrong door) "That's too bad," Mr. Hall said, opening Door 1. "You've won a goat." "But you didn't open another door yet or give me a chance to switch." "Where does it say I have to let you switch every time? I'm the master of the show."

The Confused Player

Suppose a gameshow in which the contestant is given a choice of one of three doors, behind one of which is a prize. Steps (Guaranteed): 1. The contestant must choose a door. 2 The host opens a different door which the host knows will not reveal a prize. 3. The host offers the contestant the opportunity to switch doors.

1. Why should you switch doors under these circumstances?

2. Suppose further, that to your knowledge the prize is equally likely to behind any of the three doors. Compute the probability that you win given that you switch and the probability that you win given that you stay.

3. Suppose your friend has been offered the game as described in question 2 and that your friend cannot decide whether to switch doors or not. Imagine your friend decides to flip a fair coin to decide how to deal with step 3. Heads your friend switches, Tails your friend stays. What is the probability that your friend will win? (Why) Suppose you can't bear to watch but you learn that your friend won! What is the probability that your friend's coin came up Heads given this information?

4. Suppose that to you know the prize is equally likely to behind door 1 or 2 but is twice as likely to be behind door 3 than door 1. What is the best strategy? Compute the probability that you win using this strategy.