Your Name Here

Math 105

Homework 6

For a local field E and $\alpha \in E^\times$, define $\text{ord}_E(\alpha) = m$ where $\alpha = \pi_E^m u$, $u \in U_E = O_E^\times$.

1. (4-3-18) Let E/F be an extension of local fields, $|\alpha|_E = |N_{E/F}(\alpha)|_F^{1/[E:F]}$. Show that for $\alpha \in E^\times$, $\text{ord}_F(N_{E/F}(\alpha)) = f(E/F)\,\text{ord}_E(\alpha)$.

2. (4-3-18) Let E/F be an extension of local fields. For $\alpha \in E^\times$, let $I = \alpha\mathcal{O}_E$ and define: $N_{E/F}(I) := N_{E/F}(\alpha)\mathcal{O}_F$. Show that this definition is independent of the choice of generator, and induces a homomorphism from the group of E-ideals to the group of F-ideals. Finally, show that $N_{E/F}(\mathcal{P}_E) = \mathcal{P}_F^f(E/F)$.

3. (4-3-18) Let E be a local field. Show that E contains the $(N\mathcal{P}_E - 1)$st roots of unity.

4. (4-3-18) Let E be a local field. The goal is to show that $E^\times \cong \langle \pi_E \rangle \times U_E^0 \times (1 + \mathcal{P}_E)$ where U_E^0 is the group of $N\mathcal{P}_E - 1$st roots of unity in E, and $(1 + \mathcal{P}_E)$ is the subgroup of 'principal' units of E. Start by showing that U_E^0 and $(1 + \mathcal{P}_E)$ are subgroups of $U_E = \mathcal{O}_E^\times$ with $U_E = U_E^0 \times (1 + \mathcal{P}_E)$.