Your Name Here

Math 105
Homework 3

(3-2-2) (Understanding the topology on the ideles) Let K be a number field. We know that the adeles of K, $\mathbb{A}_K$, is a topological ring, as is each factor K_ν with respect to their respective topologies. Establish the following:

1. For a non-archimedean place ν of K, show that K_ν is a topological field. Here one need only show that for $\alpha \in K_\nu^\times$, $\alpha \mapsto \alpha^{-1}$ is continuous.

2. Show that in the subspace topology on $\mathbb{A}_K^\times$, the map $\widetilde{\alpha} \mapsto \widetilde{\alpha}^{-1}$ is not continuous.

3. If instead, we embed $\mathbb{A}_K^\times$ into $\mathbb{A}_K \times \mathbb{A}_K$ via $\widetilde{\alpha} \mapsto (\widetilde{\alpha}, \widetilde{\alpha}^{-1})$, then inversion is continuous with respect to the subspace topology inherited from $\mathbb{A}_K \times \mathbb{A}_K$.