

Supplementary homework problems, due April 6, 2009

1. Let $\tau_k(n)$ denote the number of ordered k -tuples a_1, \dots, a_k of positive integers with $a_1 \cdots a_k = n$. Use induction on k to show that

$$\sum_{n \leq x} \tau_k(n) \leq \frac{x}{(k-1)!} (\log x + k - 1)^{k-1}.$$

2. Show that the set of natural numbers n with $\tau(n) > (\log n)^2$ has asymptotic density 0.
Hint: Use our result that

$$\sum_{n \leq x} \tau(n) \leq x \log x + x.$$