MATH 105: HW #3

(1) If p is a prime and m is a positive integer with $p \nmid m$, show that
$$\Phi_{mp}(x) = \frac{\Phi_m(x^p)}{\Phi_m(x)}.$$

(2) If p is a prime and m is a positive integer with $p \mid m$, show that
$$\Phi_{mp}(x) = \Phi_m(x^p).$$

(3) If p is a prime, n is a positive integer with $p \mid n$, and k is an integer with $p \mid \Phi_n(k)$, then writing $n = mp^j$, where $p \nmid m$, we have that the order of k in $(\mathbf{Z}/p\mathbf{Z})^\times$ is m.

(4) If n, k are integers with $n > 0$, then $\gcd(n, \Phi_n(k))$ is not divisible by 2 different primes.

(5) If n, k are integers with $n > 0$ and p is a prime such that the order of k in $(\mathbf{Z}/p\mathbf{Z})^\times$ is n, then $p \mid \Phi_{np^j}(k)$ for each $j = 0, 1, 2, \ldots$.

(6) Let p_n denote the nth prime and let $d_n = p_{n+1} - p_n$. Then $d_1 = 1$ and all of the other d_n's are even. Assuming the prime k-tuples conjecture, prove that each positive even integer occurs infinitely often in the sequence $(d_n)_{n\geq 1}$.