Midterm, Math 103, Fall 06

Marius Ionescu

Due: **Monday,** November 6

The first five problems are from our textbook. Good luck and start working on the midterm soon! And remember that I root for you!

1. Problem 3 on page 48.
2. Problem 4 on page 48.
3. Problem 10 on page 48.
4. Problem 13 on page 52.
5. Problem 14 on page 52.
6. Let μ be a *finitely additive* measure defined on a σ-algebra of sets, $\mathcal{U}$, contained in a space X. Suppose that $\mu(X) < \infty$ and suppose also that μ has the property that for all sequences of sets F_n in $\mathcal{U}$ such that $F_{n+1} \subset F_n$ for all n we have that $\mu(F) = \lim \mu(F_n)$, where $F = \cap_{n=1}^{\infty} F_n$. Show that μ is a measure.
7. True or false? If true, prove; if false, give a counter example. *Every bounded measurable function of the interval* $[0,1]$ *in* $\mathbb{R}$ *is the uniform limit of step functions.*
8. Let f be the function defined by the formula

   $$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

   so that f is a right-continuous nondecreasing function on $\mathbb{R}$. Let μ_f^* be the outer measure determined by f. Show that a subset of $\mathbb{R}$ is a $\bar{\mu}_f$-null set if and only if it does not contain 0. What are the $\bar{\mu}_f$-measurable sets?
9. If E is any (Lebesque-)measurable susbet of $\mathbb{R}$ such that $m(E) = 1$, where m denotes the Lebesque measure, show that there is a measurable subset $A \subset E$ such that $m(A) = 1/2$. Find another subset B of $\mathbb{R}$, measurable and containing E such that $m(B) = 2$.
10. Find a Borel set $E \subset \mathbb{R}$ such that $0 < m(E \cap J) < m(J)$ for every closed interval J. (m is the Lebesque measure).