Math 102
Foundations of Smooth Manifolds
Fall 2011
Assignment 2 (Revised Oct. 7)
Due October 12, 2011

1. (Lee 1-6) By identifying $\mathbb{R}^2$ with $\mathbb{C}$ in the usual way, we can think of the unit circle as a subset of the complex plane. An *angle function* on a subset $U \subset S^1$ is a continuous function $\theta : U \to \mathbb{R}$ such that $e^{i\theta(p)} = p$ for all $p \in U$. Show that there exists an angle function θ on an open subset $U \subset S^1$ if and only if $U \neq S^1$. For any such angle function, show that (U, θ) is a smooth coordinate chart for S^1 with its standard smooth structure.
2. Use the previous result to show that S^1 is a Lie group.
3. (Lee 1-7) For $n \in \mathbb{N}$ we let $\mathbb{CP}^n$ be the collection of 1-dimensional subspaces of the complex vector space $\mathbb{C}^{n+1}$. And let $\sim$ be the equivalence relation on $\mathbb{C}^{n+1}\backslash\{(0,\ldots,0)\}$ defined by $z \sim w$ if and only if there is a complex number $\lambda \in \mathbb{C}^*$ such that $z = \lambda w$. Let $\pi : \mathbb{C}^{n+1}\backslash\{(0,\ldots,0)\} \to \mathbb{CP}^n$ be the quotient map. Show that $\mathbb{CP}^n$ with the quotient topology is a compact $2n$-dimensional topological manifold and demonstrate how to place a smooth structure on it in a manner that is analogous to what we did in the case of $\mathbb{RP}^n$.
4. Bootby III.5.7
5. Boothby IV.2.5
6. Boothby IV.2.6