Dartmouth College

Mathematics 101

Homework 1 (due Wednesday, October 1)

In the problems below, G is a cyclic group of finite order n. While you may be inclined to do so, you may not use Lagrange's theorem nor any of its corollaries to solve these problems. You should proceed from first principles since we have not yet proven this theorem.

In particular, you may not assume that the order of an element divides the order of the group in which it lives, nor that the order of a subgroup divides the order of the group of which it is a subset.

1. Show the G contains a unique subgroup of each order d which divides n.

2. Let p be a prime and assume $p \mid n$. Show that G has exactly $p-1$ elements of order p.

3. Let H and K be subgroups of G. Show that $H \subset K$ if and only if $|H| \mid |K|$. The forward direction gives a special case of Lagrange's theorem.

4. Let $d \mid n$. Show that G has exactly d elements of exponent d.