ERRATA AND ADDENDA: *ALGEBRAIC CURVES UNIFORMIZED BY CONGRUENCE SUBGROUPS OF TRIANGLE GROUPS*

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This note gives some errata and addenda for the article *Algebraic curves uniformized by congruence subgroups of triangle groups* [1]. Thanks to Juanita Duque Rosero and Michael Schein.

(1) Before Theorem C: replace "$\mathfrak{n} \nmid 6abc$" with "$\mathfrak{n}$ coprime to $6abc$".
(2) Theorem C, Proposition 9.7: it need not follow that the projection onto many PGL_2 factors is surjective; rather, only that the image contains a dense subgroup of $\prod_{\mathfrak{p}|\mathfrak{N}} \mathrm{PSL}_2(\mathbb{Z}_{E,\mathfrak{p}})$.
(3) Lemma 5.5: should be "$(mz, m(z+1), mz(z+1))$" (replace k by z).
(4) (5.21): maps to $\mathrm{SL}_2(\mathbb{Z}_F/\mathfrak{N})/\{\pm 1\}$.
(5) Below equation (5.21): replace "Let $\mathfrak{n}$ be the prime of $E = F(a,b,c)$ below $\mathfrak{N}$" with "Let $\mathfrak{n} = \mathbb{Z}_E \cap \mathfrak{N}$ be the prime of E below $\mathfrak{N}$".
(6) Remark 5.24: sign errors crept into the second generator. The correct orthogonal elements for B are

$$1, 2\delta_a - \lambda_{2a}, (\lambda_{2a}^2 - 4)\delta_b + (\lambda_{2a}\lambda_{2b} + 2\lambda_{2c})\delta_a - (\lambda_{2a}^2\lambda_{2b} + \lambda_{2a}\lambda_{2c} - 2\lambda_{2b}),$$

not

$$1, 2\delta_a - \lambda_{2a}, (\lambda_{2a}^2 - 4)\delta_b + (\lambda_{2a}\lambda_{2b} + 2\lambda_{2c})\delta_a - (\lambda_{2a}^2\lambda_{2b} - \lambda_{2a}\lambda_{2c} + 2\lambda_{2b}).$$

In the corrected basis, we obtain the presentation:

$$B \simeq \left(\frac{\lambda_{2a}^2 - 4, -(\lambda_{2a}^2 - 4)\beta}{F}\right) \simeq \left(\frac{\lambda_{2a}^2 - 4, \beta}{F}\right)$$

when $a \neq \infty$.
(7) Proof of Theorem 9.1: the unipotent case should be allowed in the proof, when $s = \infty$. Replace the start of the middle paragraph by:

> Next, we show that orders of g_1, g_2, g_3 are $a^\sharp, b^\sharp, c^\sharp$. Let $s = a, b, c$ and write g for the corresponding element. We have $\operatorname{tr}\phi(\overline{\delta}_s) \equiv \pm\lambda_{2s} \pmod{\mathfrak{P}}$. If $g = 1$, then the image is commutative, and this possibility was just ruled out. If $s = \infty$, then since $g \neq 1$ and $\lambda_\infty = 2$ we must have g unipotent, so g has order $p = s^\sharp$.

(8) Lemma 9.8: since $\mathrm{PSL}_2(\mathbb{F}_4) \simeq \mathrm{PSL}_2(\mathbb{F}_5)$, in fact this lemma holds whenever $\#(\mathbb{Z}_F/\mathfrak{p}) \geq 4$.

References

[1] Pete L. Clark and John Voight, *Algebraic curves uniformized by congruence subgroups of triangle groups*, Trans. Amer. Math. Soc. **371** (2019), no. 1, 33–82.

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