MATH 255: ELEMENTARY NUMBER THEORY
EXAM #1 REVIEW

Problem 1. Show that if $n \equiv 3 \pmod 4$, then n cannot be the sum of the squares of two integers.

Problem 2. Show that if $b \equiv c \pmod m$, then $\gcd(b, m) = \gcd(c, m)$.

Problem 3. For which integers n is it true that $n - 2$ divides $2n^2 - 1$?

Problem 4. Show that the set $S \subset \mathbb{R}_{>0}$ of positive irrational numbers is not well-ordered.

Problem 5. Compute the inverse of 17 modulo 31, and solve the congruence $17x \equiv 10 \pmod{31}$.

Problem 6. Show that if p is an odd prime and a is a positive integer not divisible by p, then the congruence $x^2 \equiv a \pmod p$ has either no solution or exactly two incongruent solutions.

Problem 7. An integer x is randomly chosen between 100 and 1000. Estimate the probability that x is prime. *[Hint:* $\log(10) \approx 2.5$.*]*

Problem 8. Show that $\sqrt{2} + \sqrt{3}$ is irrational.

Problem 9. Show that if a and b are integers such that $\gcd(a, b) = 1$, then $\gcd(a^n, b^n) = 1$.