MATH 110: LINEAR ALGEBRA
MIDTERM #2 REVIEW

Problem 1. Mark the following statements true or false. If the statement is true, give a reason; if it is false, give a counterexample.

(a) If A, B are row-equivalent matrices, then $\det(A) = \det(B)$.
(b) Let A, B be $n \times n$ matrices such that $AB = O$. Then $\text{rk}(A) + \text{rk}(B) \leq n$.
(c) The row space of a matrix A is equal to the column space of A.
(d) If A is a matrix which is similar only to itself, then $A = I$.
(e) Let A be an $m \times n$ matrix with rank m. Then there exists an $n \times m$ matrix B such that $BA = I$, where I is the $m \times m$ identity matrix.
(f) $\det(A + A^t) = 2\det(A)$ for any matrix A.
(g) If $a \neq b$, then the $n \times n$-matrix

$$A = \begin{pmatrix} a+b & ab & 0 & \dots & 0 & 0 \\ 1 & a+b & ab & \dots & 0 & 0 \\ 0 & 1 & a+b & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a+b & ab \\ 0 & 0 & 0 & \dots & 1 & a+b \end{pmatrix}$$

has

$$\det A = \frac{a^{n+1} - b^{n+1}}{a - b}.$$

(h) If A, B, C, D are $n \times n$-matrices, then the $2n \times 2n$-matrix

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

has determinant

$$\det M = (\det A)(\det D) - \det(B)\det(C).$$

(i) Similar matrices have the same characteristic polynomial.
(j) The matrices A and A^t have the same eigenvalues.
(k) If two matrices have the same characteristic polynomial and one is diagonalizable, then so is the other.
(l) A matrix A satisfies $A^2 = I$ if and only if the only eigenvalues of A are ± 1.
(m) If $A \neq O$ is a matrix such that $A^k = O$ for some $k \geq 1$, then A is not diagonalizable.
(n) If A is an $n \times n$-matrix and $f(t)$ is a polynomial such that $f(A) = O$, then the characteristic polynomial of A divides $f(t)$.
(o) If A is a matrix with eigenvalue λ and $f(t)$ is a polynomial, then $f(\lambda)$ is an eigenvalue of $f(A)$.