MATH 052: INTRODUCTION TO PROOFS
HOMEWORK #38

Problem 6.3.15. Let $a, b, c \in \mathbb{Z}$. Show that if $\gcd(a, c) = 1$ and $c \mid ab$ then $c \mid b$.

Solution. Suppose $\gcd(a, c) = 1$ and $c \mid ab$. Since $\gcd(a, c) = 1$, there exist $x, y \in \mathbb{Z}$ such that $ax + cy = 1$. Thus $b = (ab)x + c(by)$. Since $c \mid ab$, we have $c \mid (ab)x$. Since obviously also $c \mid c(by)$, we have $c \mid ((ab)x + c(by)) = b$, as desired.

Date: 2 December2011.