

MATH 052: INTRODUCTION TO PROOFS
EXAM #1

Name _____

Problem	Score
1	
2	
3	
4	
5	

Total _____

Problem 1.

(a) True or false:

$$P \wedge (\sim Q \vee$$

is a sentential form. If it is not, make it one.

(b) Is the statement “if $2 + 2 = 3$ then the moon is made of cheese” true or false? Briefly explain.

(c) What is the contrapositive of the statement “If a function f is differentiable at $c \in \mathbb{R}$, then f is continuous at $c \in \mathbb{R}$ ”?

(d) How many elements are in the set $S = \{1, 2, 1, \{1, 2\}, \emptyset\}$?

(e) What is the negation of the statement “If you earn a passing grade on the final exam, then you will receive a passing grade for your final grade”?

Problem 2.

(a) Show that $P \Rightarrow (Q \vee R)$ and $(\sim Q) \Rightarrow ((\sim P) \vee R)$ are logically equivalent.

(b) What is a tautology?

Problem 3. Consider the following statement:

For every real number x , there is a real number y with the property that $x + y + 3 = 8$.

(a) What is the truth value of this statement? Briefly explain.

(b) Using quantifiers, express this statement in symbols.

(c) Using quantifiers, express the negation of this statement in symbols.

(d) Express the negation of this statement in words.

Problem 4. Recall that $n \in \mathbb{Z}$ is *odd* if $n = 2k + 1$ for some $k \in \mathbb{Z}$ and that $n \in \mathbb{Z}$ is *even* if $n = 2k$ for some $k \in \mathbb{Z}$.

Prove the statement:

If n is an odd integer, then $4n^2 + n - 2$ is odd.

Your proof should be clear, rigorous, and maybe even beautiful!

Problem 5.

(a) Describe the set $A = \{-2, -1, 1, 2, 3\}$ in the form $\{x \in \mathbb{Z} : P(x)\}$ where $P(x)$ is a proposition depending on the variable x .

(b) Let $A = \{1, 5, 9, 13\}$ and $B = \{3, 9, 15\}$. Determine $A \cup B$, $A \cap B$, and $A \setminus B$.

(c) Give examples of three sets A, B, C such that $B \in A$, $B \subset C$, and $A \cap C \neq \emptyset$.

(d) Which of the following sets are equal?

$$A = \{n \in \mathbb{Z} : -4 \leq n \leq 4\}$$

$$B = \{x \in \mathbb{N} : 2x + 2 = 0\}$$

$$C = \{u \in \mathbb{Z} : u^3 = 4u\}$$

$$D = \{-2, 0, 2\}$$

$$E = \{x \in \mathbb{Z} : 3x - 2 = 0\}$$