MATH 251: ABSTRACT ALGEBRA I
IN CLASS REVIEW, EXAM #2

Problem 1. True or false: if all subgroups H of a group G are normal, then G is abelian.

Problem 2. What are the cosets of the subgroup $H = \langle 4 \rangle$ in $G = \mathbb{Z}/12\mathbb{Z}$? What is the isomorphism type of the quotient G/H?

Problem 3. Let G be a finite group of odd order. How many elements of G are equal to their own inverse?

Problem 5. Let $G = Q_8$ and $H = \langle -1 \rangle$. What is the order of jH in G/H?

Problem 6. Show that the map

$$\begin{aligned} \phi : G = \mathbb{R} &\to \mathbb{C}^\times \\ x &\mapsto e^{2\pi i x} \end{aligned}$$

is a homomorphism. What are the image and kernel of ϕ? What does the First Isomorphism Theorem tell you about $G/\ker\phi$?

Date: 28 October 2011; exam 2 November 2011.