MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I
EXAM #2

Name ____________________________________

Problem	**Score**
1	
2	
3	
4	

Total ____________

Date: Friday, 19 November 2010.

Problem 1 (5 points). Mark each as true or false. Briefly justify your answer.

(a) Every bounded, infinite set A has a limit point.

(b) If $f : U \to \mathbb{R}$ is continuous and U is open then $f(U)$ is open.

(c) Let $A \subseteq \mathbb{R}$ and let S be the set of isolated points of A. Then S is closed.

(d) If $f : [a, b] \to \mathbb{R}$ is continuous and $f(a) \leq L \leq f(b)$, then there exists $c \in (a, b)$ such that $f(c) = L$.

(e) If a set $A \subseteq \mathbb{R}$ has a maximum and a minimum, then A is compact.

Problem 2 (10 points). Let

$$\begin{aligned} f &: \mathbb{R}_{\geq 0} \to \mathbb{R} \\ f(x) &= \sqrt{x}, \end{aligned}$$

where $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} : x \geq 0\}$.

(a) Prove that f is continuous, using the definition.

(b) Prove that f is uniformly continuous on $[0, 1]$.

(c) Is the derivative f' of f uniformly continuous on its domain? Justify your answer rigorously.

Problem 3 (5 points). Let $f : A \to \mathbb{R}$ be a function and let c be a limit point of A. Suppose that

$$\lim_{x \to c} f(x) = L > 0.$$

Prove that there exists a neighborhood $U \subseteq \mathbb{R}$ of c such that $f(x) > 0$ for all $x \in U \cap A$.

Problem 4 (5 points). For $a \geq 0$, define $f_a : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x^a \sin(\log(x)), & \text{if } x > 0; \\ 0, & \text{if } x \leq 0. \end{cases}$$

(a) Compute $f'(x)$ for $x \neq 0$. (You may use the familiar rules for differentiation.)

(b) Find a value $a \in \mathbb{N}$ such that f has a continuous derivative at $c = 0$, and prove rigorously that your answer is correct.