MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I
EXAM #1

Name ________________________________

Problem	**Score**
1	
2	
3	
4	
5	

Total ____________

Date: 6 October 2010.

Problem 1 (5 pts). Mark each as true or false. Briefly justify your answer.

(a) Every Cauchy sequence is bounded.

(b) Let $A \subset \mathbb{R}$ be bounded below and let $B = \{b \in \mathbb{R} : b \text{ is a lower bound for } A\}$. Then B has a maximum.

(c) The set $\{x \in \mathbb{N} : x < 17\}$ is countable.

(d) If $\sum x_n^2$ converges and $x_n \geq 0$ for all n, then $\sum x_n$ converges.

(e) Given $a, b \in \mathbb{R}$ with $a < b$, the set of irrationals in the interval (a, b) is uncountable.

Problem 2 (5 pts). Prove using the definition that

$$\lim \frac{\sqrt{n}}{\sqrt{n}+1} \to 1.$$

Problem 3 (5 pts). Let $A \subseteq \mathbb{R}$ be nonempty and bounded above. Suppose that $x \geq 0$ for all $x \in A$. Let

$$\sqrt{A} = \{\sqrt{x} : x \in A\}.$$

(a) Show that $\sup A \geq 0$. (Why does $\sup A$ exist?)

(b) Prove that

$$\sup \sqrt{A} = \sqrt{\sup A}.$$

Problem 4 (5 pts). Let (b_n) be a convergent sequence with $b_n \to b$. By the Algebraic Limit Theorem, we know that (b_n^2) converges to b^2.

Prove that $b_n^2 \to b^2$ directly using the definition. *[Hint: Use that the sequence is bounded.]*

Problem 5 (5 pts). Let (a_n) be a monotone sequence and suppose that (a_n) has a convergent subsequence. Show that (a_n) converges.