MATH 20C: FUNDAMENTALS OF CALCULUS II
WORKSHEET, DAY #40 (FINAL REVIEW)

The following is a sampling of some problems that relate to the material covered in the course. It does not cover all topics: in particular, there are no problems on probability and statistics. It is intended to help you study for the final exam, which will be on Thursday, December 18, at 3:30 p.m.

1. Multiple Choice

Problem 1. If the marginal cost function is given by $T'(x) = 3x^2+4x+3$, then a general expression for the total cost function would be:

(a) $T(x) = x^3 + 2x^2 + C$
(b) $T(x) = 6x + 4 + C$
(c) $T(x) = x^3 + 2x^2 + 3x + C$
(d) $T(x) = 3x^3 + 4x^2 + 3x + C$.

Problem 2. A function gives the rate of depreciation of a piece of farm machinery from the first through the fifth year. Interpret the area under the graph of this function.

(a) The total change in value of the machinery over the four year period.
(b) The value of the machinery at the end of the four year period.
(c) The average value of the machinery during the four year period.
(d) The total change in value of the machinery since it was purchased.

Problem 3. The definite integral $\displaystyle\int_a^b f(x)\,dx =$_____________ , where F is the antiderivative of f.

(a) $F(a) - F(b)$
(b) $f'(b) - f'(a)$
(c) $F(b - a)$
(d) $F(b) - f(a)$

Problem 4. The area enclosed by the curves $y = 1/x$ and $y = 2x$ from $x = 1$ to $x = e$ is:

(a) $2e$
(b) $e + 1$
(c) $e^2 + 1$
(d) $e^2 - 2$

Problem 5. If an integral is _____________ , then there is a finite area under the curve being graphed.

(a) divergent
(b) convergent
(c) improper
(d) rational

Date: Wednesday, December 10, 2008.

Problem 6. The function $C(x,y,z) = \dfrac{2x^2 - 4xy + zxy}{x}$ is:

(a) linear
(b) a second-order regression
(c) differential
(d) both linear and a second-order regression

Problem 7. Compute $F(-1,2,-3)$ for $F(x,y,z) = y - \sqrt{xyz - 2}$

(a) 1
(b) -5
(c) 0
(d) 2

Problem 8. Given the graph of the function $f(x,y)$, if z is set equal to a constant, we will obtain a curve resulting from a slice parallel to the ____________ -plane.

(a) xy
(b) xz
(c) yz

Problem 9. Determine f_{xy} for $f(x,y) = x\sqrt{y}$.

(a) $\dfrac{1}{2\sqrt{y}}$
(b) $\sqrt{y} + \dfrac{x}{2\sqrt{y}}$
(c) $\dfrac{1}{\sqrt{y}}$
(d) $\dfrac{1}{2x\sqrt{y}}$

Problem 10. Evaluate f_x at the point $(3,-1)$ for $f(x,y) = 2x^2y - 3xy^3$.

(a) -15
(b) 0
(c) 15
(d) -9

Problem 11. For $h(x,y) = x^2 + y^2 - 10x - 2y + 36$:

(a) $h(0,0)$ is a relative maximum.
(b) $h(5,1)$ is a relative minimum.
(c) $h(5,1)$ is a relative maximum.
(d) None of the above.

Problem 12. A rectangular piece of cardboard with length x and height y has squares of length z cut from its corners. The cardboard is then bent up to form an open-topped box. What is the volume of the box?

(a) $V(x,y,z) = xyz$
(b) $V(x,y,z) = (x-2z)(y-2z)z$
(c) $V(x,y,z) = xyz^2$
(d) $V(x,y,z) = (x-z)(y-z)z$

2. Free Response

Problem 1.

(a) Evaluate the integral $\int \frac{1}{8x}\,dx$.

(b) Evaluate the integral $\int \left(x^{1.5} - 2 + 3x^{-2}\right)\,dx$.

Problem 2. Evaluate the integral $\int \frac{\sin x}{\sqrt{1-2\cos x}}\,dx$.

Problem 3. Evaluate the integral $\int (8x-1)e^{x-4x^2}\,dx$.

Problem 4. Calculate the left Riemann sum to approximate $\int_{-1}^{1} x^2\,dx$ using $n = 6$ subintervals.

Problem 5. A worker can assemble toaster ovens at the rate of $-3t^2 + 12t + 5$ ovens per hour. How many ovens can be assembled in the first half of an 8-hour shift?

Problem 6. Evaluate the integral $\int x(x-3)^4\,dx$.

Problem 7. Determine the equation of the curve that passes through the point $(0, 0)$ and has slope xe^x.

Problem 8.

(a) Evaluate the integral $\int \frac{\cos x}{\sin^2 x}\,dx$.

(b) Find the average value of the function $f(x) = 6\sin 3x$ on the interval $[0, \pi]$.

Problem 9. Find the average value of the function $\frac{\ln x}{x^2}$ on the interval $[e, e^2]$.

Problem 10. Find the future value of the income stream $R(t) = 20t$ for $0 \le t \le 5$ at 4%.

Problem 11. Determine whether the integral $\int_2^\infty (5/x^2)\,dx$ converges or diverges. If it converges, determine its value.

Problem 12. Find the solution to the differential equation $\frac{dy}{dx}y^{-2} = 4x^3$ which has the value $y = 2$ when $x = 1$.

Problem 13. Find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y^2}$ for the function $f(x, y) = \frac{4x^2}{y} + \frac{y^2}{2x}$.

Problem 14. Classify the critical points of $f(x, y) = x^3 + y^3 - 6xy$.

Problem 15. Find the minimum value of the function $f(x, y, z) = 2xy - xz + x$ subject to the condition that $xyz = 10$.

Problem 16. A company wishes to design a rectangular box whose length, width, and double the height total to 4 inches. Find the dimensions that will maximize the volume.

Problem 17. Use Lagrange multipliers to find the maximum and minimum values for the function $f(x, y) = 2x^2 - y^2 - 18x + 2$ subject to $x^2 + y^2 = 25$.