

MATH 20C: FUNDAMENTALS OF CALCULUS II
QUIZ #6

Problem 1. Solve the differential equation $\frac{dy}{dx} = 3x^2y$ subject to the initial condition $y = 2$ when $x = 0$.

Solution. We have

$$\begin{aligned}\frac{dy}{dx} &= 3x^2y \\ \frac{dy}{y} &= 3x^2 dx \\ \int \frac{dy}{y} &= \int 3x^2 dx \\ \ln |y| &= x^3 + C \\ y &= e^{\ln |y|} = e^{x^3+C} = Ce^{x^3}.\end{aligned}$$

We substitute to find $2 = Ce^0 = C$, so $y = 2e^{x^3}$.

Problem 2.

(a) If $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2 + 1}$, compute $f(0, 0)$ and $f(a, -1)$.

Solution. We have $f(0, 0) = 0$ and $f(a, -1) = \frac{a^2 - (-1)^2}{a^2 + (-1)^2 + 1} = \frac{a^2 - 1}{a^2 + 2}$.

(b) Is the function $f(x, y, z) = x - 10000y - 0.5z + xyz$ a linear function?

Solution. No: a linear function is composed of terms of degree 1 in every variable: xyz has degree 3.

(c) Find the distance between the points $(3, 2)$ and $(5, -3)$.

Solution. The distance is $\sqrt{(5-3)^2 + (-3-2)^2} = \sqrt{2^2 + 5^2} = \sqrt{29}$.