MATH 20C: FUNDAMENTALS OF CALCULUS II QUIZ #5

Problem 1. Compute the area between $y = e^{-x}$ and y = -4x + 1 for x in [0, 1]. Include a graph. Solution. We have the following graph:



The area is

$$\int_0^1 (e^{-x} - (-4x + 1)) \, dx = (-e^{-x} + 2x^2 - x) \Big|_0^1 = (-e^{-1} + 2 - 1) - (-1) = 2 - 1/e.$$

Problem 2. Use the formula

$$FV = \int_{a}^{b} R(t)e^{r(b-t)} dt$$

to find the future value of the income stream R(t) = 200t for $0 \le t \le 10$ at 10%. Solution. We have

$$FV = \int_0^{10} 200t e^{0.1(10-t)} dt = 200 \int_0^{10} t e^{1-0.1t} dt.$$

We use integration by parts:

$$\begin{array}{c|cccc} D & I \\ \hline + & t & e^{1-0.1t} \\ - & 1 & e^{1-0.1t}/(-0.1) = -10e^{1-0.1t} \\ + \int & 0 & 100e^{1-0.1t} \end{array}$$

 So

$$\int te^{1-0.1t} dt = t(-10e^{1-0.1t}) - 100e^{1-0.1t} + C = -10(t+10)e^{1-0.1t} + C$$

hence

$$FV = 200 \left(-10(t+10)e^{1-0.1t} \right) \Big|_0^{10} = 200(-200+100e) = \$14365.$$

Date: Wednesday, October 15, 2008.