## MATH 20C: FUNDAMENTALS OF CALCULUS II QUIZ \#5

Problem 1. Compute the area between $y=e^{-x}$ and $y=-4 x+1$ for $x$ in $[0,1]$. Include a graph. Solution. We have the following graph:


The area is

$$
\int_{0}^{1}\left(e^{-x}-(-4 x+1)\right) d x=\left.\left(-e^{-x}+2 x^{2}-x\right)\right|_{0} ^{1}=\left(-e^{-1}+2-1\right)-(-1)=2-1 / e .
$$

Problem 2. Use the formula

$$
F V=\int_{a}^{b} R(t) e^{r(b-t)} d t
$$

to find the future value of the income stream $R(t)=200 t$ for $0 \leq t \leq 10$ at $10 \%$.
Solution. We have

$$
F V=\int_{0}^{10} 200 t e^{0.1(10-t)} d t=200 \int_{0}^{10} t e^{1-0.1 t} d t
$$

We use integration by parts:

|  | $D$ | $I$ |
| :---: | :---: | :---: |
| + | $t$ | $e^{1-0.1 t}$ |
| - | 1 | $e^{1-0.1 t} /(-0.1)=-10 e^{1-0.1 t}$ |
| $+\int$ | 0 | $100 e^{1-0.1 t}$ |

So

$$
\int t e^{1-0.1 t} d t=t\left(-10 e^{1-0.1 t}\right)-100 e^{1-0.1 t}+C=-10(t+10) e^{1-0.1 t}+C
$$

hence

$$
F V=\left.200\left(-10(t+10) e^{1-0.1 t}\right)\right|_{0} ^{10}=200(-200+100 e)=\$ 14365
$$

