MATH 20C: FUNDAMENTALS OF CALCULUS II
FINAL EXAM

1. Multiple Choice

(1) (b)
(2) (d)
(3) (a) $(4-(-2))/8 = 0.75$.
(4) (c) The graphs cross at the points $x = -1, 0, 1$; from $[-1, 0]$ the graph of $y = x^3$ is above $y = x$ and vice versa on $[0, 1]$.
(5) (c)
(6) (a)
(7) (b) Integration by parts gives $\int_0^1 x^2 e^x\, dx = (x^2 - 2x + 2)e^x\big|_0^1 = e - 2$.
(8) (a) The average population is $\frac{1}{30}\int_0^{30} 5e^{0.023t}\, dt = \frac{1}{6(0.023)} e^{0.023t}\big|_0^{30} = 7.2$.
(9) (b)
(10) (c) $\int_1^\infty e^{-3t}\, dt = \lim_{M\to\infty} -e^{-3t}/3 = 1/(3e^3)$.
(11) (b)
(12) (c)
(13) (b)
(14) (a)
(15) (b)
(16) (b)
(17) (d)
(18) (a)
(19) (b)
(20) (d)

2. Free Response

(1) $(\ln x)^7/7 + C$ (substitute $u = \ln x$).
(2) $1/2\big(1/(1+2) + 1/(1+3) + 1/(1+4) + 1/(1+5)\big) = 19/40$.
(3) $\dfrac{\sin(4x^3 - 3x)}{3} + C$ (substitute $u = 4x^3 - 3x$).
(4) $FV = \int_0^{10} 1200te^{0.1(10-t)}\, dt = 12000e^{-0.1t+1}(t+10)\big|_0^{10} = -12000(-20 + 10e^1) = \$86193$.
(5) $1/y\, dy = 6x\, dx$ so $\ln y = 3x^2 + C$ so $y = Ce^{3x^2} = -3e^{3x^2}$.
(6) $f_x = -ye^{-xy}$ and $f_y = -xe^{-xy}$ so $f_{xx} = y^2e^{-xy}$ and $f_{yy} = x^2e^{-xy}$; $f_{xy} = -e^{-xy} + (-y)(-x)e^{-xy} = (xy-1)e^{-xy}$ by the product rule.
(7) $f_x = 6xy - 6x = 6x(y-1) = 0$ and $f_y = 3x^2 + 3y^2 - 6y = 0$; the first equation has solutions $x = 0$ and $y = 1$, so substituting these into the second equation gives $3y(y-2) = 0$ and $3x^2 - 3 = 3(x^2-1) = 0$ so the critical points are $(0,0), (0,2), (\pm 1, 1)$.
(8) $2/x^2 \geq 0$ for $x \geq 2$ and $\int_2^\infty 2/x^2 = \lim_{M\to\infty} -2/x\big|_2^M = 1$.
(9) $g = x^2 + y^2 - 8$, so we have $f_x = 2y = \lambda g_x = \lambda(2x)$ so $y = \lambda x$ and similarly $x = \lambda y$; solving $\lambda = y/x$ and substituting gives $x = y^2/x$ so $x^2 = y^2$; hence $2x^2 = 8$ so $x = \pm 2 = y$ and the maximum value is $2xy = 2(2)(2) = 8$.
(10) We minimize the surface area $S = 2\ell h + 2\ell w + wh$ subject to $\ell hw = 32$; solving for $\ell = 32/wh$ and substituting gives $S = 64/w + 64/h + wh$; then $S_w = -64/w^2 + h = 0$ and $S_h = -64/h^2 + w = 0$; solving $h = 64/w^2$ and substituting gives $w = -64/(64/w^2)^2 = w^4/64$ so $w^3 = 64$ so $w = 4$ and $h = 64/16 = 4$ and $\ell = 32/16 = 2$, so the dimensions are $2 \times 4 \times 4$ inches.