

**MATH 251: ABSTRACT ALGEBRA I
WORKSHEET, DAY #2**

Problem 1. Mark the following true or false, and justify your answer.
Let $A, B, C \subset X$ be sets.

- (a) $A \cap B = B \cap A$;
- (b) $A \setminus B = B \setminus A$;
- (c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;
- (d) If $\#A = \#B$, then a map

$f : A \rightarrow B$ is injective $\Leftrightarrow f : A \rightarrow B$ is surjective.

Problem 2. Is the map $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = \sqrt{x}$ well-defined? Is there a way to modify f so that it is well-defined?

Problem 3. If X is a set with $\#X = 5$, then:

- (a) How many subsets does X have?
- (b) How many subsets having four elements does X have?

Problem 4. Let $f : A \rightarrow B$ be a map of sets. In this exercise, we relate properties of f to its fibers $f^{-1}(b)$ (for $b \in B$). Prove the following statements:

- (a) f is injective \Leftrightarrow for all $b \in B$, we have $\#f^{-1}(b) \leq 1$.
- (b) f is surjective \Leftrightarrow for all $b \in B$, we have $\#f^{-1}(b) \geq 1$.
- (c) f is bijective \Leftrightarrow for all $b \in B$, we have $\#f^{-1}(b) = 1$.

Problem 5. Show that $\#(A \cup B) = \#A + \#B - \#(A \cap B)$.