Errata to Morita Equivalence and Continuous-Trace C^*-algebras

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Page 8, Definition 2.1: We should have emphasized that a left inner product A-module is defined similarly except that the inner product should linear in the first variable and satisfy ${}_A\langle a \cdot x \,,\, y\rangle = a\,{}_A\langle x \,,\, y\rangle$.

Page 9, line −12: "faithful representation" should be faithful nondegenerate representation".

Page 9, line −2: "sequilinear" should be "sesquilinear".

Page 15, line 8: "cstar@group" should be omitted.

Page 15, line −4: "$\langle x \,,\, x\rangle_A \geq 0$" should be "$\langle x \,,\, x\rangle_0 \geq 0$".

Page 16, line −3: "for all $x, y \in \mathsf{X}$" should be "for all $x \in \mathsf{X}$ and $y \in \mathsf{Y}$".

Page 17, line 12: Delete "$x \in \mathsf{X}$ and".

Page 18, line −5: "X_A" should be "X_A".

Page 22, line 11–12 Replace "then standard properties of the functional calculus imply" with "then, since f is odd and can be uniformly approximated with odd polynomials, standard properties of the functional calculus imply". (To see this, notice that if f is odd and $p_n \to f$ uniformly, then $\tilde{p}_n \to f$ uniformly with $\tilde{p}_n(x) = \frac{1}{2}\big(p_n(x) - p_n(-x)\big)$.)

Page 24, line 10: "nonzero ideal A" should be "nonzero ideal in A".

Page 25, line 13: "$\lambda \in C$" should be "$\lambda \in \mathbb{C}$".

Page 28, line 6: Change "monomorphism" to "a monomorphism".

Page 36 lines 7 and 12: The reference "(2.23)" should be "(2.25)".

Page 42, line 12: Replace "some mild smoothness conditions" with "some smoothness and growth conditions".

Page 42, line −13: "${}_A\langle x, y\rangle(t)$" should be "${}_{C_0(T,\mathcal{K}(\mathcal{H}))}\langle x, y\rangle(t)$".

Page 44, line −3: replace "for all $x \in \mathsf{X}$" with "for all $x \in \mathsf{X}_0$".

Page 45, line 1: Replace "X" by "X_0".

Page 45, line −10: Replace "$A - B$-imprimitivity bimodule" with "$A - B$-pre-imprimitivity bimodule".

Page 52, line 13: "(x, c)" should be "(x, a)".

Page 56, line 10: Replace "$a \in J$, $b \in K$" with "$a \in K$, $b \in J$".

Page 76, line 8: "for the the" should be "for the".

Page 77, line −5: "$\tilde{\phi}(B^n(\mathbf{U}, \mathcal{S}))$" should be "$\tilde{\phi}(B^n(\mathbf{U}, \mathcal{R}))$".

Page 83, line 8: "$N_x \cap \overline{W_{i_1,\dots,i_n}}$" should be replaced by "$N_x \setminus \overline{W_{i_1,\dots,i_n}}$".

Page 84, line −10: "$H^1(X, \mathcal{S})$" should be $H^1(X, \underline{\mathbb{Z}})$".

Page 86, line 13: "$H^1(X, \mathcal{S})$" should be $H^1(X, \underline{\mathbb{Z}})$".

Page 87, lines 6 and 7: A number of changes should be made to the last paragraph of Example 4.39. On line 6, "$(-\frac{1}{2}, \frac{3}{2})/\sim$" should be replaced by "$(-\frac{1}{3}, 1)/\sim$". On line 7, "sets $U_1 := \dots$ are" should be replaced by "sets $U_1 := (-\frac{1}{3}, \frac{1}{3})$, $U_2 := (0, \frac{2}{3})$ and $U_3 := (\frac{1}{3}, 1)$ are".

Page 87, line 20: In Lemma 4.40, "f_*" should be replaced by "f^*", etc.

Page 89, line 8: "cohomology group" should be "cohomology groups".

Page 92, line −2: "$h(x) \in p^{-1}(U)$ has" should be "$x \in p^{-1}(X)$, $h(x)$ has"

Page 93, line −10: "Mobius@Möbius" should be "Möbius".

Page 94, line 7: replace the union with $\bigcup_{n\in\mathbb{Z}}(x_0 + n, x_1 + n)$.

Page 94, line 9: $h(x + n) = (\exp 2\pi i x, n)$.

Page 97, line 5 "H^n" should be "H^1".

Page 100, line 8: Remark 4.64 is (slightly) inaccurate. The principal bundles in [77] are exactly the free G-spaces satisfying (c). These spaces are called *Cartan G-spaces* in [121]. If the orbit space (or base space of the bundle in [77]) is Hausdorff, then these spaces coincide with the free and proper G-spaces [121, Theorem 1.2.9]. In general, a free Cartan G-space need not be a proper G-space — see the example following Proposition 1.1.4 in [121]. In view of this, the second sentence of the remark should read "If G acts freely and satisfies (b) and (c), then G automatically acts properly; thus the locally compact principal bundles over Hausdorff spaces in [77] correspond to the free and proper G-spaces".

Page 103, lines 1 and 5: Replace "$(1-t,1]$" by "$(t,1]$".

Page 111, line 5 "$=$" should be "$\cong$".

Page 118, line −8: "in Dauns-Hofmann" should be "in the Dauns-Hofmann".

Page 120, line -5**:** Replace "$q^F_{\mathsf{Y}} \circ \phi^F$" with "$q^F_{\mathsf{Y}} \circ \phi$".

Page 124, lines 18 & 21: Replace "$C_0(X)$" with "$C(X)$".

Page 127, line 9: "$B^{F_{ij}}$" should be "$C(F_{ij})$".

Page 127, line 18: "$\delta^2(A)$" should be "$\delta(A)$".

Page 130, line 13: Replace "$= a^{F_{ij}} p_i^{F_{ij}}$" by "$= a^{F_{ij}} (v_{ij}^{F_{ij}})^*$".

Page 130, lines −11−−1: The proof of Lemma 5.28(b) (i.e., the last paragraph on page 130) should be replaced by "Note B" on page 7 of these errata.

Page 131, line 18: Both α and β must be $C_0(T)$-linear.

Page 138, line 10: "$\{U_{ij}\}$ should be "$\{U_i\}$".

Page 140, line 11: "$[\pi_{i,t)}]$" should be "$[\pi_{(i,t)}]$".

Page 157, line 10: "induces an isomorphism".

Page 161, line −12 The induced homomorphism f^* is also defined in Lemma 4.40.

Page 163, §6.3 The definition of $\operatorname{Ind}_G^X(A,\alpha)$ really doesn't make much sense unless X/G is Hausdorff. Fortunately, X/G is Hausdorff in all our applications.

Page 164, line 17: "$\operatorname{Ind}_G^K(A,\alpha)$" should be "$\operatorname{Ind}_G^K(A,\beta)$".

Page 175, line −11: The formula "$f^*(s) := \Delta(s^{-1})f(s^{-1})^*$" should be "$f^*(s) := \Delta(s^{-1})\alpha_s(f(s^{-1}))^*$."

Page 177, line −1: Replace "$\operatorname{Aut} A$" with "$UM(A)$".

Page 178, line −14: "(B,B,β)" should be "(B,G,β)".

Page 188, line 6: "$f : G \to A$" should be "$f : G^n \to A$".

Page 189, line 8–9: If the G-action on A is not trivial, then it may not be the case that the product of Haar measure on A with the left Haar measure on G is a left-invariant measure on E_ω. However, the product of the Haar measure on A with a right Haar measure on G is *right*-invariant on E_ω.

The Mackey and Weil result from [99, Theorem 7.1] still applies, and E_ω has a locally compact topology compatible with its Borel structure.[1]

Page 197, line −3: Replace "$H^2(X;\mathbb{Z})^*$" with "$H^0(T;\mathbb{Z})^*$".

Page 203, line −11: "only if $\sigma(a) \subset [0,\infty)$" should be replaced by "only if $a = a^*$ and $\sigma(a) \subset [0,\infty)$".

Page 204, line 8: "and $\rho \in S(A)$" should be "and ρ is a state on A".

Page 207, line 5: Replace "$\notin B(\lambda;R)$." with "$\notin B(\lambda;R)$, where $B(\lambda;R) = \{\, \tau \in \mathbb{C} : |\tau - \lambda| \le R \,\}$.

Page 210, line 14: Replace "$\psi(a)$" by "$\psi(a^*a)$".

Page 214, line 4: "thus $S \in \hat{A}$ is open in $\hat{A}$ if and only if ... in $\operatorname{Prim} A$." should be replaced by "$S \subset \operatorname{Prim} A$ is open if and only if $\{\, \pi \in \hat{A} : \ker \pi \in S \,\}$ is open in $\hat{A}$."

Page 214, line −14: "$t \in \mathbb{T}$" should be "$t \in T$ ".

Page 214, line −12: "an isomorphism".

Page 222, line 7: "an invariant infinite-dimensional subspace".

Hooptedoodle A.51 on page 232: Comment: in a recent annoncement (July 2001), Nik Weaver has issued a preprint giving an example of a prime ideal which is not primitive.

Page 236, line −8: "bilinear from $A \odot B$" should be "bilinear from $A \times B$".

Page 239, Lemma B.6: I can't follow the last paragraph of the proof. However, it suffices to prove the lemma with the *additional hypothesis* that A has an identity. Then the last paragraph of the proof can be replaced with the following obseravation:

Lemma Suppose that A is a C^*-algebra with identity and that C is a subset of the state space of A such that for all self-adjoint a, $\|a\| = \sup\{\, |\rho(a)| : \rho \in C \,\}$. Then the convex hull of C is weak-$*$ dense in the state space of A.

[1] Although not strictly necessary, it might be interesting to note that we can exhibit a left invariant measure on E_ω directly. Let $\sigma : G \to (0,\infty)$ be the continuous homomorphism determined by

$$\sigma(t) \int_A g(t \cdot a)\, d\mu_A(a) = \int_A g(a)\, d\mu_A(a).$$

Then we get a left-invariant integral on $E_\omega = A \times G$ by

$$I(f) := \int_A \int_A f(a,t)\sigma(t)^{-1}\, d\mu_A(a)\, d\mu_G(t).$$

Proof. Let D be the closed convex hull of C. The functional calculus implies that a self-adjoint element a is positive if and only if $\|a\|1_A - a$ has norm bounded by $\|a\|$. Thus

$$a = a^* \text{ and } \rho(a) \geq 0 \text{ for all } \rho \in C \text{ implies that } a \geq 0. \tag{1}$$

If the convex hull of C is not dense, then there is a state τ which is not in D. Thus τ has a convex neighborhood disjoint from D and Lemma A.40 implies that there is an $a \in A$ and an $\alpha \in \mathbf{R}$ such that

$$\operatorname{Re} \tau(a) < \alpha \leq \operatorname{Re} \rho(a) \quad \text{for all } \rho \in C.$$

Since $\rho(a^*) = \overline{\rho(a)}$ for any state ρ, we can replace a by $a_0 := \frac{1}{2}(a + a^*)$ so that

$$\tau(a_0) < \alpha \leq \rho(a_0) \quad \text{for all } \rho \in C.$$

It follows from (1) that $a_0 - \alpha 1_A \geq 0$. But then, since τ is positive, $\tau(a_0) \geq \alpha$. This is a contradiction and completes the proof.

Page 239, line −6: Since we added the hypothesis that A have a unit to Lemma B.6, it no longer applies directly. However, if $\widetilde{\mathfrak{A}}$ is the C^*-subalgebra generated by $\mathfrak{A}$ and the identity, then we can apply Lemma B.6 to $\widetilde{\mathfrak{A}}$ with the observation that every state of $\mathfrak{A}$ extends to a state on $\widetilde{\mathfrak{A}}$ by Lemma A.6.

Page 245, line 2: Replace "isomorphism ψ" with "isomorphism ϕ".

Page 252, line 16: Replace "$B \to M(B \otimes_{\max} D)$" with "$C \to M(B \otimes_{\max} D)$".

Page 262, line 1: Replace "Every C^*-algebra" with "Every CCR C^*-algebra".

Page 271, lines 5–14: The argument proving that we can reduce to the case were G is σ-compact is badly flawed. A replacement for the first paragraph of the proof in given on page 6 of these errata as "Note A". Our proof and the result itself should be compared to [55, Lemma 2.53].

Page 273, line 9–10: Replace "By multiplying a Bruhat …on supp(f)" by "By multiplying a Bruhat approximate cross-section by a function in $C_c^+(G/H)$ which is identically one on supp f".

Page 278, line 17: There is a missing "$d\mu(s)$" in Equation (C.15).

Page 281, line 10: "cstar@group" should be omitted.

Page 287, line −5: "correspondence".

Page 288, line 4: "$f \cdot b =$ " should be "$f \cdot b(s) =$ ".

Page 288, line 9: Both "$C(G/H)$"'s should be "$C_0(G/H)$".

Page 290, line −5: "$\|F\|^2$" should be $\|F\|_\infty^2$".

Page 290, line −3: "$\|F\|_\infty$" should be "$\|F\|_\infty^2$".

Page 298, line 10: Replace "$W(f \otimes h)$" with"$W(f \otimes h)(r)$".

Page 303, line −9: "An inductive limit" should be "A direct limit".

Page 304, line 5: Replace "G/H" with "G/F".

Page 304, lines 4 & 7: Comment: we used "0" to denote the identity element of any group. Which group should be clear from context.

Page 305, line 9: Replace "locally convex space M" with "locally convex topological vector space M".

Page 307, line 11: Replace "$f - f_0 \in W$" with "$f - f_0 \in cW$".

Note A: The first paragraph of the proof of Proposition C.1 should be replaced with the following.

We claim it suffices to prove the result when when G is σ-compact. Let G_0 be a σ-compact open subgroup of G (such as that generated by any compact neighbourhood of e in G). Let I be a set of double coset representatives for $G_0\backslash G/H$, so that G is the disjoint union

$$\bigcup_{a\in I} G_0 aH.$$

Since G_0 is open, each double coset G_0aH is open, and since $\overline{G_0aH} \subset G_0^2aH = G_0aH$, each double coset is also closed.[2] For each $a \in I$, let $H^a := aHa^{-1}$ and let ν^a be the Haar measure[3] on H^a given by

$$\int_{H^a} f(\omega)\, d\nu^a(\omega) := \int_H f(ata^{-1})\, d\nu(t) \quad \text{for } f \in C_c(H^a).$$

Let $H_0^a := H^a \cap G_0$. Since H_0^a is an open subgroup of H^a, the restriction of ν^a to H_0^a is a Haar measure ν_0^a on H_0^a. Since G_0 is σ-compact and H_0^a is a closed subgroup, we may assume that there is a Bruhat approximate cross section b_a for G_0 over H_0^a with respect to ν_0^a. Since G_0 is closed and open, we can extend b_a to a bounded continuous function on G by letting it be identically zero off G_0. Suppose that $s \in G_0$ and $t \in H^a$. Then $st \in G_0$ implies $t \in H^a \cap G_0 = H_0^a$. Since b_a vanishes off G_0 and is approximate section for G_0 over H_0^a,

$$\int_{H^a} b_a(st)\, d\nu^a(t) = \int_{H_0^a} b_a(st)\, d\nu_0^a(t) = 1 \quad \text{for all } s \in G_0. \tag{2}$$

Since the double cosets are both closed and open, we can define a bounded continuous function on G by

$$b(s) := b_a(sa^{-1}) \quad \text{if } s \in G_0aH \text{ for } a \in I.$$

[2]If V is a symmetric neighbourhood of e in G and $A \subset G$, then $\overline{VA} \subset V^2A$. To see this, let $x \in \overline{VA}$. Then Vx is a neighbourhood of x and must meet VA. Thus $x \in V^2A$.

[3]Note that we can have $H^a = H^b$ without having $\nu^a = \nu^b$.

We claim that b is a Bruhat approximate cross section for G over H. We first check the integral condition. Let $x \in G$. Then there is a $a \in I$ such that $x = sah$ with $s \in G_0$ and $h \in H$. Then, in view of (2), we have

$$\int_H b(xt)\,d\nu(t) = \int_H b_a(sahta^{-1})\,d\nu(t) = \int_{H^a} b_a(s\omega)\,d\nu^a(\omega) = 1.$$

Now let C be a compact set in G. Since CH meets at most finitely many double cosets, it suffices to assume that $C \subset G_0aH$ for some $a \in I$ and prove that $\operatorname{supp} b \cap CH$ is compact. But $\{ G_0ah \}_{h\in H}$ is an open cover of C. Thus

$$C = \bigcup_{i=1}^n C_i a h_i$$

for compact sets $C_i \subset G_0$ and $h_i \in H$. Therefore

$$\operatorname{supp} b \cap CH = \bigcup_{i=1}^n \operatorname{supp} b \cap C_i aH.$$

If $s \in C_i$, $h \in H$ and $b(sah) \neq 0$, then $b_a(saha^{-1}) \neq 0$. This implies $saha^{-1} \in G_0$ and $aha^{-1} \in H_0^a$. That is, $sah \in C_iH_0^a \cdot a$. It follows that

$$\operatorname{supp} b \cap CH \subset \bigcup_{i=1}^n \bigl(\operatorname{supp} b_a \cap C_iH_0^a\bigr) \cdot a.$$

Our assumptions on b_a imply that the right-hand side is compact. It follows that b is the desired section, and it suffices to treat the σ-compact case as claimed.

Note B: This material replaces the last paragraph of the proof of Lemma 5.28 on page 130. (There is a problem with the partition of unity argument.)

Let $\{ F_i \}$, $\{ U_i \}$, $\{ \mathsf{X}_i \}$ and g_{ij} be as in Proposition 5.24. As in the proof of Proposition 5.15, given $t \in U_i$, we can find a $x_i \in \mathsf{X}_i$ such that $\langle x_i, x_i\rangle_{C(F_i)} \equiv 1$ near t. Thus be refining the cover $\{ U_i \}$ if necessary, we can assume that $\langle x_i, x_i\rangle_{C(F_i)} \equiv 1$ on all of F_i. Now let $p_i \in A$ be such that $p_i^{F_i} =_{A^{F_i}} \langle x_i, x_i\rangle$. Then for each $t \in F_i$, Lemma 5.16 implies that $p_i(t)$ is a rank-one projection. A similar argument shows that any $v_{ij} \in A$ satisfying

$$v_{ij}^{F_{ij}} =_{A^{F_{ij}}} \langle x_i^{F_{ij}}, g_{ij}(x_j^{F_{ij}})\rangle$$

has the properties required in (5.5).