2003 Kemeny Lecture Series

Finsler metrics of constant flag curvature

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102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

Finsler geometry is a generalization of Riemannian geometry in which one generalizes from having a smoothly varying (positive definite) inner product on the tangent space at each point to having a smoothly varying strictly convex Banach norm on the tangent space at each point. Many problems in the calculus of variations for curves are naturally formulated as geodesic problems on Finsler manifolds and, indeed, this is where the main interest in this geometry arises.

Thus, associated to each Finsler geometry, there is its family of geodesics. The first natural invariant one defines in Finsler geometry is the invariant that governs the variation of these geodesics, i.e., the Jacobi fields. A Finsler manifold is said to have *constant flag curvature* c if its Jacobi operator along any geodesic is conjugate to that along a geodesic in a Riemannian space form of constant sectional curvature c. In contrast to the Riemannian case, a Finsler manifold of constant flag curvature does not have to be homogeneous or even Riemannian.

In this talk, I will review the basic about Finsler manifolds and their local invariants and discuss what is known about the existence and generality, both local and global, of Finsler metrics of constant flag curvature. Among the more recent interesting results have been the determination of these structures in dimension 2 and, via some recent results of LeBrun and Mason, the proof that the Riemannian round 2-sphere is the only reversible Finsler metric of constant flag curvature $+1$ on the 2-sphere.

This talk should be accessible to specialists only.