

2003 Kemeny Lecture Series

New Zoll metrics on the 2-sphere

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102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

It is classical that, on the standard round 2-sphere in 3-space, the geodesics are the great circles and, hence, are all closed. This is a very special situation since, for the 'generic' metric on the 2-sphere, almost none of the geodesics close.

A metric on the 2-sphere for which all of the geodesics close is nowadays called a *Zoll metric*, after Otto Zoll, who about a century ago, used some ideas of Darboux to construct an explicit family of metrics on the 2-sphere with the property that all of the geodesics of any member of this family are closed. Each of Zoll's examples has an axis of symmetry, but Funk conjectured (and, much later, Victor Guillemin proved) that the Zoll metrics that are 'near' the round metric in a suitable sense can be parametrized by the odd functions on the 2-sphere, thus providing a potentially enormous family of Zoll metrics on the 2-sphere. However, other than examples with an axis of symmetry, none of these have been found explicitly before now.

In this talk, I will review the now-classical story outlined above and will use some ideas that go back to Jacobi (who showed that the geodesics on the ellipsoid form a completely integrable system) to construct some new explicit examples of Zoll metrics, in fact, ones with no rotational symmetry.