Dartmouth College Department of Mathematics
Math 81/111 Abstract Algebra Syllabus
Winter Term 2020

Instructor:	Professor Asher Auel	**Lecture:**	Kemeny 343
Office:	Kemeny 339	**Time:**	MWF 10:10 – 11:15 am Th 12:15 – 1:15 pm X-hour
Text:	*Abstract Algebra, 3rd edition*, David S. Dummit and Richard M. Foote John Wiley & Sons, ISBN-13: 978-0-471-43334-7.		
Web-site:	http://math.dartmouth.edu/~auel/courses/81w20/		

Introduction: The main object of study in Galois theory are roots of single variable polynomials. Many ancient civilizations (Babylonian, Egyptian, Greek, Chinese, Indian, Persian) knew how to solve quadratic equations. Today, most middle schoolers memorize the "quadratic formula" by heart. While various incomplete methods for solving cubic equations were developed in the ancient world, a general "cubic formula" (as well as a "quartic formula") was not known until the 16th century Italian school. It was proven by Ruffini and Abel, that the roots of the general quintic polynomial could not be solvable in terms of nested roots. Galois theory provides a satisfactory explanation for this, as well as to the unsolvability (proved independently in the 19th century) of several classical problems concerning compass and straight-edge constructions (e.g., trisecting the angle, doubling the cube, squaring the circle). More generally, Galois theory is all about symmetries of the roots of polynomials. An essential concept is the field extension generated by the roots of a polynomial. The philosophy of Galois theory has also impacted other branches of higher mathematics (Lie groups, topology, number theory, algebraic geometry, differential equations).

This course will provide a rigorous proof-based modern treatment of the main results of field theory and Galois theory. The main topics covered will be irreducibility of polynomials, Gauss's lemma, field extensions, minimal polynomials, separability, field automorphisms, Galois groups and correspondence, constructions with ruler and straight-edge, theory of finite fields. Some advanced topics, such as infinite Galois theory and Galois cohomology, will be included. The grading in Math 81/111 is very focused on precision and correct details. Problem sets will consist of a mix of computational and proof-based problems.

Grading: Final grades will be based on weekly homework, a takehome midterm exam, and a final exam. While significant emphasis is placed on exams, completing your weekly homework will be crucial to your success on the exams and in the course.

Homework	40 %
Takehome Midterm	25 %
Final Exam (09 Mar)	35 %

Exams: The takehome midterm exam will take place over a week in February. The final exam will take place 8:00 am – 12:00 pm on Monday 09 March. The use of internet resources during the takehome midterm exam, or any electronic devices during the final exam, will not be allowed.

Homework: There will be weekly problem sets assigned, due in class on Friday. The problem sets will be posted on the course web-site syllabus page the week before they are due. Your lowest problem set score of at least 50% will be dropped from your final grade calculation. Unless a valid excuse is prearranged, late problem sets turned in within one week of the deadline will be worth only 50% of its score. After one week, problem sets will not be accepted. If you know in advance that you will be unable to submit your homework at the correct time and place, you must make special arrangements ahead of time.

Consider (as you would for any other class) the pieces of paper you turn in as a final copy: written neatly and straight across the page, on clean paper, stapled together, with nice margins, lots of space, and well organized. You might consider taking the opportunity to learn LaTeX.

Group work, honestly: Working with other people on mathematics is not only allowable, but is highly encouraged and fun. You may work with anyone (e.g., other students in the course, students not in the course, tutors) on the rough draft of your problem sets. If done right, you'll learn the material better and more efficiently working in groups. The golden rule is:

> You may work with anyone on *solving* your homework problems,
> but you must *write* up your final draft by yourself.

Writing up the final draft is as important a process as figuring out the problems on scratch paper with your friends. Mathematical writing is very idiosyncratic—it is easy to tell if papers have been copied from others or from the internet—just don't do it! You will not learn, and will engage in academic dishonesty, by copying solutions! Also, if you work with people on a particular assignment, you *must list your collaborators at the top of the paper*, as well as any resources (e.g., Wikipedia) used beside the text book. All claims in your solutions must be fully and rigorously justified. You are free to cite results from lecture, from the book, and within reason, from anywhere else (as long as it doesn't make the problem trivial). Make the process fun, transparent, and honest.

Prerequisites: Previous exposure to linear and abstract algebra (Math 24 and Math 71) is required. If you had Math 22 and/or Math 31, please consult with me about enrolling in the course.

Topics covered: Subject to change.

(1) Review of polynomial rings. Euclidean division. Euclidean algorithm for calculating gcd. Euclidean ⇒ PID ⇒ UFD. Polynomials as functions. Roots as linear factors.

(2) Fundamental Theorem of Algebra. Irreducibility of polynomials. Gauss's Lemma. Eisenstein Criterion. Reduction modulo p.

(3) Field extensions. Degree. Simple extensions. Algebraic and transcendental extensions. Minimal polynomial. Field automorphisms.

(4) Ruler and compass constructions. Solvable numbers.

(5) Separability. Perfect fields.

(6) Linear independence of characters and embeddings. Normal closure.

(7) Galois extensions. The Galois group of a polynomial. Galois correspondence.

(8) Primitive element theorem. Solvable fields. Solving for roots with radicals.

(9) Finite fields. Cyclic extensions.

(10) Elementary symmetric polynomials. General polynomial. Discriminant.

(11) Solving equations of degree at most 4. Calculating Galois groups. Algebraic proof of the Fundamental Theorem of Arithmetic.

(12) (Advanced topics) Newton polytopes. Infinite Galois theory. Galois cohomology. Hilbert theorem 90. Norm and trace. Integrality.

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