Dartmouth College Department of Mathematics
Math 71 Algebra
Fall 2021

Problem Set # 0 (due via Canvas upload by 5 pm, Wednesday, September 22)

Notation: If S is a set of elements (numbers, rabbits, ...) then the notation "$s \in S$" means "s is an element of the set S." If T is another set, then the notation "$T \subseteq S$" means "every element of T is an element of S" or "T is a **subset** of S." We can specify a subset $T \subset S$ by conditions on the elements of S, e.g., if S is the set of rectangles, then the subset of squares is $\{s \in S \mid \text{all sides of } s \text{ have the same length}\}$. If S and T are sets, then a **function** or **map** $f : S \to T$ from S to T is the a rule that associates to each element $s \in S$, an element $f(s) \in T$.

Reading: DF 0.1–0.3, 1.1.

Problems: (Starred* problems are required.)

1. DF 0.1 Exercises 5, 6, 7*.
DF 0.2 Exercises 7, 10*, 11.
DF 0.3 Exercises 3, 4, 5, 6, 7, 8, 12*, 13*, 14*.

2. DF 1.1 Exercises 6*, 7, 8, 12, 15*, 16*, 17*, 20, 22*, 23*, 25* (Hint. Consider $(xy)^2$), 31*, 32, 34.

3. Let G be a group and $g \in G$.
(a) Prove that if $ga = a$ for all $a \in G$ (or that $ag = a$ for all $a \in G$) then g is the identity element.
(b) Prove that if $gg = g$ then g is the identity element.

4. The set of invertible $n \times n$ real matrices is a group $\mathrm{GL}_n(\mathbb{R})$ with the operation of matrix multiplication, called the **general linear group**. Consider the following elements of $\mathrm{GL}_2(\mathbb{R})$:

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Show that A and B have finite order (compute their orders) but that AB has infinite order. This shows that the order of a product is not necessarily the product of the orders! (Though see Problem Set 1 for an instance when this does hold.)

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